Homework #19
Stat 202

**5.43 Should you use the binomial distribution?** In each of the following situations, is it reasonable to use a binomial distribution for the random variable $X$? Give reasons for your answer in each case. If a binomial distribution applies, give the values of $n$ and $p$.

**(a)** A poll of 200 college students asks whether or not you are usually irritable in the morning. $X$ is the number who reply that they are usually irritable in the morning.

**(b)** You toss a fair coin until a head appears. $X$ is the count of the number of tosses that you make.

**(c)** Most calls made at random by sample surveys don't succeed in talking with a live person. Of calls to New York City, only one-twelfth succeed. A survey calls 500 randomly selected numbers in New York City. $X$ is the number of times that a live person is reached.

**(d)** You deal 10 cards from a shuffled deck and count the number $X$ of black cards.

**5.45 Typographic errors.** Typographic errors in a text are either nonword errors (as when "the" is typed as "teh") or word errors that result in a real but incorrect word. Spell-checking software will catch nonword errors but not word errors. Human proofreaders catch 70% of word errors. You ask a fellow student to proofread an essay in which you have deliberately made 10 word errors.

**(a)** If the student matches the usual 70% rate, what is the distribution of the number of errors caught? What is the distribution of the number of errors missed?

**(b)** Missing 4 or more out of 10 errors seems a poor performance. What is the probability that a proofreader who catches 70% of word errors misses 4 or more out of 10?

**5.47 Typographic errors.** Return to the proofreading setting of Exercise 5.45.

**(a)** What is the mean number of errors caught? What is the mean number of errors missed? You see that these two means must add to 10, the total number of errors.

**(b)** What is the standard deviation $\sigma$ of the number of errors caught?

**(c)** Suppose that a proofreader catches 90% of word errors, so that $p = 0.9$. What is $\sigma$ in this case? What is $\sigma$ if $p = 0.99$? What happens to the standard deviation of a binomial distribution as the probability of a success gets close to 1?

**5.53 Inheritance of blood types.** Children inherit their blood type from their parents, with probabilities that reflect the parents' genetic makeup. Children of Juan and Maria each have probability 1/4 of having blood type A and inherit independently of each other. Juan and Maria plan to have 4 children; let $X$ be the number who have blood type A.

**(a)** What are $n$ and $p$ in the binomial distribution of $X$?

**(b)** Find the probability of each possible value of $X$, and draw a probability histogram for this distribution.

**(c)** Find the mean number of children with type A blood, and mark the location of the mean on your probability histogram.

**5.61 A test for ESP.** In a test for ESP (extrasensory perception), the experimenter looks at cards that are hidden from the subject. Each card contains either a star, a circle, a wave, or a square. As the experimenter looks at each of 20 cards in turn, the subject names the shape on the card.

**(a)** If a subject simply guesses the shape on each card, what is the probability of a successful guess on a single card? Because the cards are independent, the count of successes in 20 cards has a binomial distribution.

**(b)** What is the probability that a subject correctly guesses at least 10 of the 20 shapes?

**(c)** In many repetitions of this experiment with a subject who is guessing, how many cards will the subject guess correctly on the average? What is the standard deviation of the number of correct guesses?

**(d)** A standard ESP deck actually contains 25 cards. There are five different shapes, each of which appears on 5 cards. The subject knows that the deck has this makeup. Is a binomial model still appropriate for the count of correct guesses in one pass through this deck? If so, what are $n$ and $p$? If not, why not?

**5.62 Admitting students to college.** A selective college would like to have an entering class of 950 students. Because not all students who are offered admission accept, the college admits more than 950 students. Past experience shows that about 75% of the students admitted will accept. The college decides to admit 1200 students. Assuming that students make their decisions independently, the number who accept has the $B(1200, 0.75)$ distribution. If this number is less than 950, the college will admit students from its waiting list.

**(a)** What are the mean and the standard deviation of the number $X$ of students who accept?

**(b)** Use the Normal approximation to find the probability that at least 800 students accept.

**(c)** The college does not want more than 950 students. What is the probability that more than 950 will accept?

**(d)** If the college decides to increase the number of admission offers to 1300, what is the probability that more than 950 will accept?

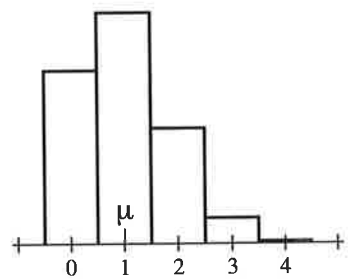
**5.43.** **(a)** A $B(200, p)$ distribution seems reasonable for this setting (even though we do not know what $p$ is). **(b)** This setting is not binomial; there is no fixed value of $n$. **(c)** A $B(500, 1/12)$ distribution seems appropriate for this setting. **(d)** This is not binomial, because separate cards are not independent.

**5.45.** **(a)** $C$, the number caught, is $B(10, 0.7)$. $M$, the number missed, is $B(10, 0.3)$. **(b)** Referring to Table C, we find $P(M \geq 4) = 0.2001 + 0.1029 + 0.0368 + 0.0090 + 0.0014 + 0.0001 = 0.3503$ (software: 0.3504).

**5.47.** **(a)** The mean of $C$ is $(10)(0.7) = 7$ errors caught; for $M$ the mean is $(10)(0.3) = 3$ errors missed. **(b)** The standard deviation of $C$ (or $M$) is $\sigma = \sqrt{(10)(0.7)(0.3)} \doteq 1.4491$ errors. **(c)** With $p = 0.9$, $\sigma = \sqrt{(10)(0.9)(0.1)} \doteq 0.9487$ errors; with $p = 0.99$, $\sigma \doteq 0.3146$ errors. $\sigma$ decreases toward 0 as $p$ approaches 1.

**5.53.** **(a)** $n = 4$ and $p = 1/4 = 0.25$. **(b)** The distribution is below; the histogram is on the right. **(c)** $\mu = np = 1$.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | .3164 | .4219 | .2109 | .0469 | .0039 |




**5.61.** **(a)** $p = 1/4 = 0.25$. **(b)** $P(X \geq 10) = 0.0139$. **(c)** $\mu = np = 5$ and $\sigma = \sqrt{np(1-p)} = \sqrt{3.75} \doteq 1.9365$ successes. **(d)** No: The trials would not be independent because the subject may alter his/her guessing strategy based on this information.

**5.62.** **(a)** $\mu = (1200)(0.75) = 900$ and $\sigma = \sqrt{225} = 15$ students. **(b)** $P(X \geq 800) \doteq P(Z \geq -6.67) = 1$ (essentially). **(c)** $P(X \geq 951) \doteq P(Z \geq 3.4) = 0.0003$. **(d)** With $n = 1300$, $P(X \geq 951) \doteq P(Z \geq -1.54) = 0.9382$. Other answers are shown in the table on the right.

| | | *Continuity correction* | |
|---|---|---|---|
| Table Normal | Software Normal | Table Normal | Software Normal |
| 0.9382 | 0.9379 | 0.9418 | 0.9417 |