Homework #10
Stat 202

4.34 PINs. The personal identification numbers (PINs) for automatic teller machines usually consist of four digits. You notice that most of your PINs have at least one 0, and you wonder if the issuers use lots of 0s to make the numbers easy to remember. Suppose that PINs are assigned at random, so that all four-digit numbers are equally likely.

(a) How many possible PINs are there?

(b) What is the probability that a PIN assigned at random has at least one 0?

4.35 Universal blood donors. People with type O-negative blood are universal donors. That is, any patient can receive a transfusion of O-negative blood. Only 7% of the American population have O-negative blood. If 10 people appear at random to give blood, what is the probability that at least 1 of them is a universal donor?

4.36 Disappearing Internet sites. Internet sites often vanish or move, so that references to them can't be followed. In fact, 13% of Internet sites referenced in papers in major scientific journals are lost within two years after publication.[10] If a paper contains seven Internet references, what is the probability that all seven are still good two years later? What specific assumptions did you make in order to calculate this probability?

4.49 What's wrong? In each of the following scenarios, there is something wrong. Describe what is wrong and give a reason for your answer.

(a) The probabilities for a discrete statistic always add to one.

(b) A continuous random variable can take any value between zero and one.

(c) Normal distributions are discrete random variables.

4.53 Discrete or continuous. In each of the following situations decide if the random variable is discrete or continuous and give a reason for your answer.

(a) Your Web page has five different links and a user can click on one of the links or can leave the page. You record the length of time that a user spends on the Web page before clicking one of the links or leaving the page.

(b) The number of hits on your Web page.

(c) The yearly income of a visitor to your Web page.

4.54 Texas hold 'em. The game of Texas hold 'em starts with each player receiving two cards. Here is the probability distribution for the number of aces in two-card hands:

Number of aces	0	1	2
Probability	0.8507	0.1448	0.0045

(a) Verify that this assignment of probabilities satisfies the requirement that the sum of the probabilities for a discrete distribution must be 1.

(b) Make a probability histogram for this distribution.

(c) What is the probability that a hand contains at least one ace? Show two different ways to calculate this probability.

4.60 Foreign-born residents of California. The Census Bureau reports that 27% of California residents are foreign-born. Suppose that you choose three Californians at random, so that each has probability 0.27 of being foreign-born and the three are independent of each other. Let the random variable W be the number of foreign-born people you chose.

(a) What are the possible values of W?

(b) Look at your three people in order. There are eight possible arrangements of foreign (F) and domestic (D) birth. For example, FFD means the first two are foreign-born and the third is not. All eight arrangements are equally likely. What is the probability of each one?

(c) What is the value of W for each arrangement in (b)? What is the probability of each possible value of W? (This is the distribution of a Yes/No response for an SRS of size 3. In principle, the same idea works for an SRS of any size.)

4.61 Uniform random numbers. Let X be a random number between 0 and 1 produced by the idealized uniform random number generator described in Example 4.25 and Figure 4.9. Find the following probabilities:

(a) $P(X < 0.6)$

(b) $P(X \leq 0.6)$

(c) What important fact about continuous random variables does comparing your answers to parts (a) and (b) illustrate?

4.62 Find the probabilities. Let the random variable X be a random number with the uniform density curve in Figure 4.9. Find the following probabilities:

(a) $P(X \geq 0.30)$

(b) $P(X = 0.30)$

(c) $P(0.30 < X < 1.30)$

(d) $P(0.20 \leq X \leq 0.25 \text{ or } 0.7 \leq X \leq 0.9)$

(e) The probability that X is not in the interval 0.4 to 0.7.

Solutions

4.34. **(a)** There are $10^4 = 10{,}000$ possible PINs (0000 through 9999).* **(b)** The probability that a PIN has *no* 0s is 0.9^4 (because there are 9^4 PINs that can be made from the nine nonzero digits), so the probability of at least one 0 is $1 - 0.9^4 = 0.3439$.

*If we assume that PINs cannot have leading 0s, then there are only 9000 possible codes (1000–9999), and the probability of at least one 0 is $1 - \frac{9^4}{9000} = 0.271$.

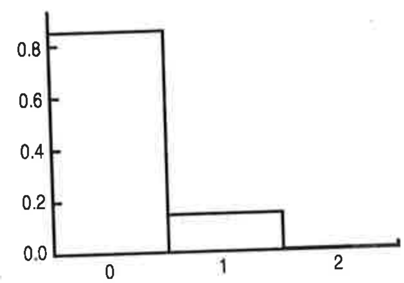
4.35. $P(\text{none are O-negative}) = (1 - 0.07)^{10} \doteq 0.4840$, so $P(\text{at least one is O-negative}) \doteq 1 - 0.4840 = 0.5160$.

4.36. If we assume that each site is independent of the others (and that they can be considered as a random sample from the collection of sites referenced in scientific journals), then $P(\text{all seven are still good}) = 0.87^7 \doteq 0.3773$.

4.49. **(a)** The probabilities for a discrete *random variable* always add to one. **(b)** Continuous random variables can take values from any interval, not just 0 to 1. **(c)** A Normal random variable is continuous. (Also, a distribution is *associated with* a random variable, but "distribution" and "random variable" are not the same things.)

4.53. **(a)** See also the solution to Exercise 4.22. If we view this time as being measured to any degree of accuracy, it is continuous; if it is rounded, it is discrete. **(b)** A count like this must be a whole number, so it is discrete. **(c)** Incomes—whether given in dollars and cents, or rounded to the nearest dollar—are discrete. (However, it is often useful to treat such variables as continuous.)

4.54. **(a)** $0.8507 + 0.1448 + 0.0045 = 1$. **(b)** Histogram on the right. (The third bar is so short that it blends in with the horizontal axis.) **(c)** $P(\text{at least one ace}) = 0.1493$, which can be computed either as $0.1448 + 0.0045$ or $1 - 0.8507$.



4.60. **(a)** W can be 0, 1, 2, or 3. **(b)** See the top two lines of the table below. **(c)** The distribution is given in the bottom two lines of the table. For example, $P(W = 0) = (0.73)(0.73)(0.73) \doteq 0.3890$, and in the same way, $P(W = 3) = 0.27^3 \doteq 0.1597$. For $P(W = 1)$, note that each of the three arrangements that give $W = 1$ have probability $(0.73)(0.73)(0.27) = 0.143883$, so $P(W = 1) = 3(0.143883) \doteq 0.4316$. Similarly, $P(W = 2) = 3(0.73)(0.27)(0.27) \doteq 0.1597$.

Arrangement	DDD	DDF	DFD	FDD	FFD	FDF	DFF	FFF
Probability	0.3890	0.1439	0.1439	0.1439	0.0532	0.0532	0.0532	0.0197
Value of W	0		1			2		3
Probability	0.3890		0.4316			0.1597		0.0197

4.61. **(a)** $P(X < 0.6) = 0.6$. **(b)** $P(X \le 0.6) = 0.6$. **(c)** For continuous random variables, "equal to" has no effect on the probability; that is, $P(X = c) = 0$ for any value of c.

4.62. **(a)** $P(X \ge 0.30) = 0.7$. **(b)** $P(X = 0.30) = 0$. **(c)** $P(0.30 < X < 1.30) = P(0.30 < X < 1) = 0.7$. **(d)** $P(0.20 \le X \le 0.25 \text{ or } 0.7 \le X \le 0.9) = 0.05 + 0.2 = 0.25$. **(e)** $P(\text{not } [0.4 \le X \le 0.7]) = 1 - P(0.4 \le X \le 0.7) = 1 - 0.3 = 0.7$.