(Optional) Homework 23
Stat 202

6.104 Make a recommendation. Your manager has asked you to review a research proposal that includes a section on sample size justification. A careful reading of this section indicates that the power is 35% for detecting an effect that you would consider important. Write a short report for your manager explaining what this means and make a recommendation on whether or not this study should be run.

6.105 Explain power and sample size. Two studies are identical in all respects except for the sample sizes. Consider the power versus a particular sample size. Will the study with the larger sample size have more power or less power than the one with the smaller sample size? Explain your answer in terms that could be understood by someone with very little knowledge of statistics.

6.106 Power for a different alternative. The power for a two-sided test of the null hypothesis $\mu = 0$ versus the alternative $\mu = 3$ is 0.82. What is the power versus the alternative $\mu = -3$? Explain your answer.

6.107 More on the power for a different alternative. A one-sided test of the null hypothesis $\mu = 20$ versus the alternative $\mu = 30$ has power equal to 0.6. Will the power for the alternative $\mu = 40$ be higher or lower than 0.6? Draw a picture and use this to explain your answer.

6.110 Planning another test to compare cholesterol levels. Example 6.15 (page 371) gives a test of a hypothesis about the cholesterol level for female sedentary undergraduates based on a sample of size $n = 71$. The hypotheses are

$$H_0: \mu = 168$$
$$H_a: \mu \neq 168$$

While the result was not statistically significant, it did provide some evidence that the mean was larger than 168. Thus, the researcher plans to recruit another sample of sedentary females but this time using a one-sided alternative. He plans to obtain $n = 70$ subjects and wonders if this sample size gives him adequate power to detect an increase of 5 mg/dl to $\mu = 173$.

(a) Given $\alpha = 0.05$, for what values of z will he reject the null hypothesis?

(b) Using $\sigma = 27$ and $\mu = 168$, for what values of $\bar{x}$ will he reject H_0?

(c) Using $\sigma = 27$ and $\mu = 173$, what is the probability $\bar{x}$ will fall in the region defined in part (b)?

(d) Does it appear he has adequate power for a sample size of $n = 70$? Or does he need to find ways to increase it? Explain your answer.

6.111 Power of the mean SAT score test. Example 6.16 (page 372) gives a test of a hypothesis about the SAT scores of California high school students based on an SRS of 500 students. The hypotheses are

$$H_0: \mu = 450$$
$$H_a: \mu > 450$$

Assume that the population standard deviation is $\sigma = 100$. The test rejects H_0 at the 1% level of significance when $z \geq 2.326$, where

$$z = \frac{\bar{x} - 450}{100/\sqrt{500}}$$

Is this test sufficiently sensitive to usually detect an increase of 10 points in the population mean SAT score? Answer this question by calculating the power of the test against the alternative $\mu = 460$.

6.112 Choose the appropriate distribution. You must decide which of two discrete distributions a random variable X has. We will call the distributions p_0 and p_1. Here are the probabilities they assign to the values x of X:

x	0	1	2	3	4	5	6
p_0	0.1	0.1	0.2	0.1	0.1	0.1	0.3
p_1	0.2	0.2	0.2	0.1	0.1	0.1	0.1

You have a single observation on X and wish to test

$$H_0: p_0 \text{ is correct}$$
$$H_a: p_1 \text{ is correct}$$

One possible decision procedure is to reject H_0 only if $X \leq 2$.

(a) Find the probability of a Type I error, that is, the probability that you reject H_0 when p_0 is the correct distribution.

(b) Find the probability of a Type II error.

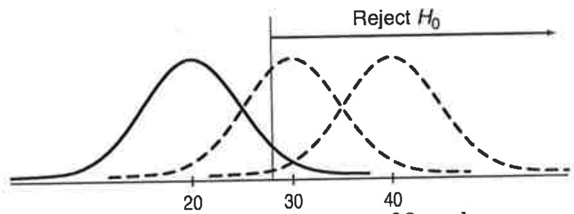
Solutions

6.104. The power of this study is far lower than what is generally desired—for example, it is well below the "80% standard" mentioned in the text. For the specified effect, 35% power means that, if the effect is present, we will only detect it 35% of the time. With such a small chance of detecting an important difference, the study should probably not be run (unless the sample size is increased to give sufficiently high power).

6.105. A larger sample gives more information and therefore gives a better chance of detecting a given alternative; that is, larger samples give more power.

6.106. The power for $\mu = -3$ is 0.82—the same as the power for $\mu = 3$—because both alternatives are an equal distance from the null value of μ. (The symmetry of two-sided tests with the Normal distribution means that we only need to consider the size of the difference, not the direction.)

6.107. The power for $\mu = 40$ will be higher than 0.6, because larger differences are easier to detect. The picture on the right shows one way to illustrate this (assuming Normal distributions): The solid curve (centered at 20) is the distribution under the null hypothesis, and the two dashed curves represent the alternatives $\mu = 30$ and $\mu = 40$. The shaded region under the middle curve is the power against $\mu = 30$; that is, that shaded region is 60% of the area under that curve. The power against $\mu = 40$ would be the corresponding area under the rightmost curve, which would clearly be greater than 0.6.



6.110. (a) For the alternative H_a: $\mu > 168$, we reject H_0 at the 5% significance level if $z > 1.645$. **(b)** $\frac{\bar{x} - 168}{27/\sqrt{70}} > 1.645$ when $\bar{x} > 168 + 1.645 \cdot \frac{27}{\sqrt{70}} \doteq 173.31$. **(c)** When $\mu = 173$, the probability of rejecting H_0 is

$$P(\bar{x} > 173.31) = P\left(\frac{\bar{x} - 173}{27/\sqrt{70}} > \frac{173.31 - 173}{27/\sqrt{70}}\right) \doteq P(Z > 0.10) = 0.4602.$$

(d) The power of this test is not up to the 80% standard suggested in the text; he should collect a larger sample.

Note: *Software gives a slightly different answer for the power in part (c), but the conclusion in part (d) is the same. To achieve 80% power against* $\mu = 173$, *we need* $n = 180$.

6.111. We reject H_0 when $z > 2.326$, which is equivalent to $\bar{x} > 450 + 2.326 \cdot \frac{100}{\sqrt{500}} \doteq 460.4$, so the power against $\mu = 460$ is

$$\begin{aligned} P(\text{reject } H_0 \text{ when } \mu = 460) &= P(\bar{x} > 460.4 \text{ when } \mu = 460) \\ &= P\left(Z > \frac{460.4 - 460}{100/\sqrt{500}}\right) \doteq P(Z > 0.09) = 0.4641. \end{aligned}$$

This is quite a bit less than the "80% power" standard.

6.112. (a) $P(\text{Type I error}) = P(X \leq 2 \text{ when the distribution is } p_0) = 0.4$.
(b) $P(\text{Type II error}) = P(X > 2 \text{ when the distribution is } p_1) = 0.4$.