

Stat 202  
Fall 2014  
Final Exam  
12/11/14  
Time Limit: 150 Minutes

Name (Print): Answers

This exam contains 8 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. A calculator is OK as long as it has no internet. You may use the browser on the lab computer (but not your computer) to access StatCrunch. No other computer use is allowed.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	15	
2	10	
3	10	
4	15	
5	10	
6	10	
7	10	
8	10	
9	10	
Total:	100	

1. (15 points) The results of the first two exams are in. Below are the scores of a random sample of 7 students—out of a total of 544 students across 20 sections. Each student in the sample has been given a three letter nickname.

Name	Amy	Joe	Sue	Jan	Dan	Eva	Mia
1 <sup>st</sup> exam, $S_1$	90	87	78	82	64	77	95
2 <sup>nd</sup> exam, $S_2$	95	90	79	90	80	85	99

You observe that all students in the sample did better on the second exam, but you had no reason to expect, ahead of time, that scores on either test would be higher or lower.

- (a) (5 points) Report a p-value to assess the significance of the results with respect to evidence that the mean test on the second exam (over all 544 students) is higher than the mean test score on the first exam (over all 544 students).

Use ~~two~~ two sided alternative because  
paired t-test  $H_0: \mu_D = 0$   
p value = 0.0136

t-stats →  
paired →  
~~with data~~

- (b) (5 points) If, overall, students performed equally well on the two exams, the results reported above might seem a little unusual. How unusual would these results be? Specifically what fraction of the time would you see results as extreme or more extreme than observed in the table above if students performed equally on the two exams?

This is the p value  
probability = 0.0136

- (c) (5 points) Are the results significant at the  $\alpha = 0.01$  significance level?

No  $0.0136 > 0.01$

2. (10 points) A college student, Klia, claims she can identify the maker of different brands of Cola. A skeptical bartender tests her claim with 100 drinks, randomly selected from the following six brands: Coke, Pepsi, RC Cola, Virgil's Cola, Giant Brand Cola, and Kirkland Cola. The Klia gets the drink right only 25 times. Another student, Joe, knows he can't identify colas, but wants free drinks, so he makes the same claim as Klia. Each time the bartender delivers Joe a drink, Joe rolls a die and lets the result determine his choice. Joe expects to make the correct selection, on average,  $1/6$  of the time. Klia didn't do much better than Joe, but is her performance significantly better than a chance performance? (i) Perform a test of significance and report the p-value. (ii) Is the result significant at the traditional significance level?

proportion stats  $\rightarrow$  one sample

# successes = 25

# observations = 100

$H_0: p = 1/6 = 0.166667$

(i) pvalue = 0.0253

(ii) Significant at 0.05 level traditional level

3. (10 points) The average GPA for the student body at Hypothetical University (HU) is reported to be 3.2. Eight randomly selected left-handers have the following GPAs: 3.3, 3.5, 3.1, 3.9, 3.0, 3.4, 3.7, 3.6. From this sample, do we have evidence to support the claim that left-handers HU get different grades than the average student? (i) Construct a test of significance and report the p-value. (ii) Is the result significant at the traditional significance level?

T-stats  $\rightarrow$  one sample

pvalue = 0.0615

Not significant at traditional level

4. (15 points) In a random sample of 55 men and 45 women, 36 of the men preferred blue over green whereas only 20 of the women preferred blue over green.
- (a) (5 points) Report a 96% confidence interval for the difference in the proportions of men and women preferring blue over green.

Stat  $\rightarrow$  proportion stats  $\rightarrow$  with summary

Sample 1 (men) 36 successes  
55 observations

Sample 2 (women) 20 successes  
45 observations

Confidence interval Level 0.96  
(0.00889, 0.41170)

- (b) (5 points) The confidence interval determines the result of a test of significance concerning the equality of the proportions. What would (i) the null hypothesis and (ii) the level of significance of this test be?

$$H_0: p_1 - p_2 = 0$$

$$\alpha = 4\%$$

- (c) (5 points) (i) Would the test from part (b) above reject the null hypothesis? (ii) Describe how you know the result of this test of significance from the confidence interval.

Reject the null hypothesis because

0 is not within interval.

5. (10 points) According to Wikipedia, Paul, the famous octopus from the 2010 World Cup, correctly predicted the result of 7 out of 7 matches, prompting to people to believe that Paul had special powers of prediction. What is the probability that an octopus without special powers will get 7 out of 7 predictions right, assuming that all the predictions are independent, and each winning team has a 50-50 chance (i.e. probability .5) of being selected by the octopus?

Stat → Calculators → Binomial

$$n = 7 \quad p = 0.50$$

$$P(X=7) = 0.0078$$

$$\text{probability} = 0.0078$$

(Note: nothing to answer here: some people suspect that the assumptions above did not hold for Paul: he may have had a preference for the German flag used for the predictions. Most of Paul's correct predictions involved German wins—although Paul did correctly predict the two German losses, and he correctly predicted the result of the final game which did not involve Germany. Purely informational. Nothing is being asked here.)

6. (10 points) Given the genetic makeup of hypothetical parents, the probability of bearing a son (versus a daughter) is  $1/2$  and the probability of having a child with brown hair is  $3/4$ . And given this hypothetical mix of parental genes, the only other possible hair color for the child is blonde. Assume the sex and hair color of the child are independent. What is the probability of the parents bearing a child who is a blonde girl?

$$P(\text{girl}) = 1/2$$

$$P(\text{blonde}) = 1/4$$

because they are independent

$$\begin{aligned} P(\text{girl and blonde}) &= P(\text{girl}) P(\text{blonde}) \\ &= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \end{aligned}$$

7. (10 points) Random variable  $X$  has mean 2 and standard deviation 3. Random variable  $Y$  has mean 3 and standard deviation 4. The random variables  $X$  and  $Y$  are independent and  $Z = X - Y$ . What is the mean and standard deviation of the random variable  $Z$ ?

$$\mu_z = \mu_x - \mu_y = 2 - 3 = -1$$

$$\sigma_z^2 = \sigma_x^2 + \sigma_y^2 = 9 + 16 = 25$$

$$\sigma_z = 5$$

$$\text{mean } z = -1$$

$$\text{std dev } z = 5$$

8. (10 points) A casino is considering how much to charge to play a game that pays the player \$5 with probability 0.2, \$50 with probability 0.02, and \$500 dollars with probability 0.002, and otherwise pays nothing. What is the minimum the casino should charge to expect to break even (on average, pay out equal to what they charge)?

mean amount casino pays

$$= 5 \times .2 + 50 \times .02 + 500 \times .002 + 0$$

$$= 3$$

Casino should charge \$3 per game

9. (10 points) In a population of 3000 individuals, the weights of the people in the population have mean 175 pounds and standard deviation 10 pounds. Forty-nine random samples of 16 people in this population are chosen. For each of the 49 samples chosen, a statistic is calculated: the mean of the weight of the people in the sample (i.e. the sample mean).

(a) (5 points) What is the mean of these 49 sample means, each sample with 16 persons?

$$\text{mean} = 175$$

(b) (5 points) What is the standard deviation of these 49 sample means, each sample with 16 persons?

$$\begin{aligned}\text{std dev} &= \frac{\text{POP std dev}}{\sqrt{\text{sample size}}} \\ &= \frac{10}{\sqrt{16}} = \frac{10}{4} = \frac{5}{2} \\ &= 2.5\end{aligned}$$