Homework 11 Don At least 3 Solutions at end

SECTION 4.3 Summary

A **random variable** is a variable taking numerical values determined by the outcome of a random phenomenon. The **probability distribution** of a random variable *X* tells us what the possible values of *X* are and how probabilities are assigned to those values.

A random variable *X* and its distribution can be **discrete** or **continuous**.

A **discrete random variable** has finitely many possible values. The probability distribution assigns each of these values a probability between 0 and 1

such that the sum of all the probabilities is exactly 1. The probability of any event is the sum of the probabilities of all the values that make up the event.

A **continuous random variable** takes all values in some interval of numbers. A **density curve** describes the probability distribution of a continuous random variable. The probability of any event is the area under the curve and above the values that make up the event.

Normal distributions are one type of continuous probability distribution. You can picture a probability distribution by drawing a **probability histogram** in the discrete case or by graphing the density curve in the continuous case.

SECTION 4.3 Exercises

For Exercise 4.46, see page 250; for Exercise 4.47, see page 252; and for Exercise 4.48, see page 255.

- **4.49 What's wrong?** In each of the following scenarios, there is something wrong. Describe what is wrong and give a reason for your answer.
- (a) The probabilities for a discrete statistic always add to one.
- **(b)** A continuous random variable can take any value between zero and one.
- (c) Normal distributions are discrete random variables.
- **4.50** Use of Twitter. Suppose that the population proportion of Internet users who say that they use Twitter or another service to post updates about themselves or to see updates about others is 19%. ¹⁶ Think about selecting random samples from a population in which 19% are Twitter users.
- (a) Describe the sample space for selecting a single person.
- (b) If you select three people, describe the sample space.
- (c) Using the results of (b), define the sample space for the random variable that expresses the number of Twitter users in the sample of size 3.
- (d) What information is contained in the sample space for part (b) that is not contained in the sample space for part (c)? Do you think this information is important? Explain your answer.
- **4.51** Use of Twitter. Find the probabilities for parts (a), (b), and (c) of the previous exercise.
- **4.52** Households and families in government data. In government data, a household consists of all occupants of a dwelling unit, while a family consists of two or more persons who live together and are related by blood or

marriage. So all families form households, but some households are not families. Here are the distributions of household size and of family size in the United States:

Number of persons	1	2	3	4	5	6	7
Household probability	0.27	0.33	0.16	0.14	0.06	0.03	0.01
Family probability	0	0.44	0.22	0.20	0.09	0.03	0.02

Make probability histograms for these two discrete distributions, using the same scales. What are the most important differences between the sizes of households and families?

- **4.53 Discrete or continuous.** In each of the following situations decide if the random variable is discrete or continuous and give a reason for your answer.
- (a) Your Web page has five different links and a user can click on one of the links or can leave the page. You record the length of time that a user spends on the Web page before clicking one of the links or leaving the page.
- (b) The number of hits on your Web page.
- (c) The yearly income of a visitor to your Web page.
- **4.54 Texas hold 'em.** The game of Texas hold 'em starts with each player receiving two cards. Here is the probability distribution for the number of aces in two-card hands:

Number of aces	0	1	2
Probability	0.8507	0.1448	0.0045

- (a) Verify that this assignment of probabilities satisfies the requirement that the sum of the probabilities for a discrete distribution must be 1.
- (b) Make a probability histogram for this distribution.

- **(c)** What is the probability that a hand contains at least one ace? Show two different ways to calculate this probability.
- **4.55 Spell-checking software.** Spell-checking software catches "nonword errors," which result in a string of letters that is not a word, as when "the" is typed as "teh." When undergraduates are asked to write a 250-word essay (without spell-checking), the number *X* of nonword errors has the following distribution:

Value of X	0	1	2	3	4
Probability	0.1	0.3	0.3	0,2	0.1

- (a) Sketch the probability distribution for this random variable.
- **(b)** Write the event "at least one nonword error" in terms of *X*. What is the probability of this event?
- (c) Describe the event $X \le 2$ in words. What is its probability? What is the probability that X < 2?
- **4.56** Length of human pregnancies. The length of human pregnancies from conception to birth varies according to a distribution that is approximately Normal with mean 266 days and standard deviation 16 days. Call the length of a randomly chosen pregnancy *Y*.
- (a) Make a sketch of the density curve for this random variable.
- **(b)** What is $P(Y \le 280)$?
- **4.57** Tossing two dice. Some games of chance rely on tossing two dice. Each die has six faces, marked with $1, 2, \ldots, 6$ spots called pips. The dice used in casinos are carefully balanced so that each face is equally likely to come up. When two dice are tossed, each of the 36 possible pairs of faces is equally likely to come up. The outcome of interest to a gambler is the sum of the pips on the two up-faces. Call this random variable X.
- (a) Write down all 36 possible pairs of faces.
- **(b)** If all pairs have the same probability, what must be the probability of each pair?
- **(c)** Write the value of *X* next to each pair of faces and use this information with the result of (b) to give the probability distribution of *X*. Draw a probability histogram to display the distribution.
- (d) One bet available in craps wins if a 7 or an 11 comes up on the next roll of two dice. What is the probability of rolling a 7 or an 11 on the next roll?
- (e) Several bets in craps lose if a 7 is rolled. If any outcome other than 7 occurs, these bets either win or

- continue to the next roll. What is the probability that anything other than a 7 is rolled?
- 4.58 Nonstandard dice. Nonstandard dice can produce interesting distributions of outcomes. You have two balanced, six-sided dice. One is a standard die, with faces having 1, 2, 3, 4, 5, and 6 spots. The other die has three faces with 0 spots and three faces with 6 spots. Find the probability distribution for the total number of spots *Y* on the up-faces when you roll these two dice.
- 4.59 Dungeons & Dragons. Role-playing games like Dungeons & Dragons use many different types of dice, usually having either 4, 6, 8, 10, 12, or 20 sides. Roll a balanced 8-sided die and a balanced 6-sided die and add the spots on the up-faces. Call the sum X. What is the probability distribution of the random variable X?
- **4.60** Foreign-born residents of California. The Census Bureau reports that 27% of California residents are foreign-born. Suppose that you choose three Californians at random, so that each has probability 0.27 of being foreign-born and the three are independent of each other. Let the random variable *W* be the number of foreign-born people you chose.
- (a) What are the possible values of W?
- (b) Look at your three people in order. There are eight possible arrangements of foreign (F) and domestic (D) birth. For example, FFD means the first two are foreign-born and the third is not. All eight arrangements are equally likely. What is the probability of each one?
- (c) What is the value of W for each arrangement in (b)? What is the probability of each possible value of W? (This is the distribution of a Yes/No response for an SRS of size 3. In principle, the same idea works for an SRS of any size.)
- **4.61 Uniform random numbers.** Let *X* be a random number between 0 and 1 produced by the idealized uniform random number generator described in Example 4.25 and Figure 4.9. Find the following probabilities:
- (a) P(X < 0.6)
- **(b)** $P(X \le 0.6)$
- **(c)** What important fact about continuous random variables does comparing your answers to parts (a) and (b) illustrate?
- **4.62** Find the probabilities. Let the random variable X be a random number with the uniform density curve in Figure 4.9. Find the following probabilities:
- (a) $P(X \ge 0.30)$
- **(b)** P(X = 0.30)

- (c) P(0.30 < X < 1.30)
- (d) $P(0.20 \le X \le 0.25 \text{ or } 0.7 \le X \le 0.9)$
- (e) The probability that X is not in the interval 0.4 to 0.7.
- 4.63 Uniform numbers between 0 and 2. Many random number generators allow users to specify the range of the random numbers to be produced. Suppose that you specify that the range is to be all numbers between 0 and 2. Call the random number generated *Y*. Then the density curve of the random variable *Y* has constant height between 0 and 2, and height 0 elsewhere.
- (a) What is the height of the density curve between 0 and 2? Draw a graph of the density curve.
- **(b)** Use your graph from (a) and the fact that probability is area under the curve to find $P(Y \le 1.6)$.
- (c) Find P(0.5 < Y < 1.7).
- (d) Find $P(Y \ge 0.95)$.

4.64 The sum of two uniform random numbers.

Generate *two* random numbers between 0 and 1 and take *Y* to be their sum. Then *Y* is a continuous random variable that can take any value between 0 and 2. The density curve of *Y* is the triangle shown in Figure 4.12.

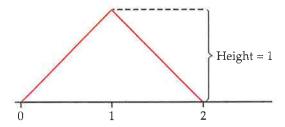


FIGURE 4.12 The density curve for the sum *Y* of two random numbers, for Exercise 4.64.

- (a) Verify by geometry that the area under this curve is 1.
- **(b)** What is the probability that *Y* is less than 1? (Sketch the density curve, shade the area that represents the probability, then find that area. Do this for (c) also.)
- **(c)** What is the probability that *Y* is greater than 0.6?
- **4.65** How many close friends? How many close friends do you have? Suppose that the number of close friends adults claim to have varies from person to person with mean $\mu = 9$ and standard deviation $\sigma = 2.4$. An opinion poll asks this question of an SRS of 1100 adults. We will see in the next chapter that in this situation the sample mean response \overline{x} has approximately the Normal distribution with mean 9 and standard deviation 0.0724. What is $P(8 \le \overline{x} \le 10)$, the probability that the statistic \overline{x} estimates the parameter μ to within ± 1 ?

4.66 Normal approximation for a sample proportion.

A sample survey contacted an SRS of 700 registered voters in Oregon shortly after an election and asked respondents whether they had voted. Voter records show that 56% of registered voters had actually voted. We will see in the next chapter that in this situation the proportion \hat{p} of the sample who voted has approximately the Normal distribution with mean $\mu=0.56$ and standard deviation $\sigma=0.019$.

- (a) If the respondents answer truthfully, what is $P(0.52 \le \hat{p} \le 0.60)$? This is the probability that the statistic \hat{p} estimates the parameter 0.56 within plus or minus 0.04.
- **(b)** In fact, 72% of the respondents said they had voted $(\hat{p} = 0.72)$. If respondents answer truthfully, what is $P(\hat{p} \geq 0.72)$? This probability is so small that it is good evidence that some people who did not vote claimed that they did vote.

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4.42. (a) Hannah and Jacob's children can have alleles AA, BB, or AB, so they can have blood type A, B, or AB. (The table on the right shows the possible combinations.) (b) Either note that the four combinations in the table are equally likely, or compute:

$$P(\text{type A}) = P(\text{A from Hannah and A from Jacob}) = P(A_H)P(A_J) = 0.5^2 = 0.25$$

 $P(\text{type B}) = P(\text{B from Hannah and B from Jacob}) = P(B_H)P(B_J) = 0.5^2 = 0.25$
 $P(\text{type AB}) = P(A_H)P(B_J) + P(B_H)P(A_J) = 2 \cdot 0.25 = 0.5$

- **4.43.** (a) Nancy and David's children can have alleles BB, BO, or OO, so they can have blood type B or O. (The table on the right shows the possible combinations.) (b) Either note that the four combinations in the table are equally likely or compute $P(\text{type O}) = P(\text{O from Nancy and O from David}) = 0.5^2 = 0.25$ and P(type B) = 1 P(type O) = 0.75.
- **4.44.** Any child of Jennifer and José has a 50% chance of being type A (alleles AA or AO), and each child inherits alleles independently of other children, so $P(\text{both are type A}) = 0.5^2 = 0.25$. For one child, we have P(type A) = 0.5 and P(type AB) = P(type B) = 0.25, so that $P(\text{both have same type}) = 0.5^2 + 0.25^2 + 0.25^2 = 0.375 = \frac{3}{8}$.
- **4.45.** (a) Any child of Jasmine and Joshua has an equal (1/4) chance of having blood type AB, A, B, or O (see the allele combinations in the table). Therefore, P(type O) = 0.25. (b) $P(\text{all three have type O}) = 0.25^3 = \begin{array}{c} A & O \\ \hline AB & BO \\ AO & OO \\ \hline \end{array}$ $0.015625 = \frac{1}{64}$. $P(\text{first has type O}, \text{ next two do not}) = 0.25 \cdot 0.75^2 = 0.140625 = \frac{9}{64}$.
- **4.46.** P(grade of D or F) = P(X = 0 or X = 1) = 0.05 + 0.04 = 0.09.
- **4.47.** If H is the number of heads, then the distribution of H is as given on the right. P(H=0), the probability of two tails was previously computed in Exercise 4.17.

Value of H	0	1	2
Probabilities	1/4	1/2	1/4

- **4.48.** P(0.1 < X < 0.4) = 0.3.
- **4.49.** (a) The probabilities for a discrete *random variable* always add to one. (b) Continuous random variables can take values from any interval, not just 0 to 1. (c) A Normal random variable is continuous. (Also, a distribution is *associated with* a random variable, but "distribution" and "random variable" are not the same things.)

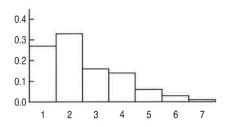
4.50. (a) If T is the event that a person uses Twitter, we can write the sample space as $\{T, T^c\}$. (b) There are various ways to express this; one would be

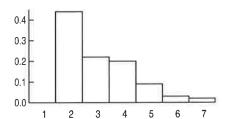
$$\{TTT, TTT^c, TT^cT, T^cTT, TT^cT^c, T^cTT^c, T^cT^cT, T^cT^cT^c\}.$$

- (c) For this random variable (call it X), the sample space is $\{0, 1, 2, 3\}$. (d) The sample space in part (b) reveals which of the three people use Twitter. This may or may not be important information; it depends on what questions we wish to ask about our sample.
- **4.51.** (a) Based on the information from Exercise 4.50, along with the complement rule, P(T) = 0.19 and $P(T^c) = 0.81$. (b) Use the multiplication rule for independent events; for example, $P(TTT) = 0.19^3 = 0.0069$, $P(TTT^c) = (0.19^2)(0.81) = 0.0292$, $P(TT^cT^c) = (0.19)(0.81^2) = 0.1247$, and $P(T^cT^cT^c) = 0.81^3 = 0.5314$. (c) Add up the probabilities from (b) that correspond to each value of X.

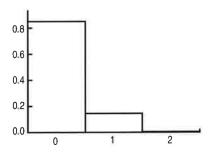
Outcome	TTT	TTT^c	$TT^{c}T$	$T^{c}TT$	T^cT^cT	T^cTT^c	TT^cT^c	$T^cT^cT^c$
Probability	0.0069	0.0292	0.0292	0.0292	0.1247	0.1247	0.1247	0.5314
Value of X	0		1			2		3
Probability	0.0069		0.0877			0.3740		0.5314

4.52. The two histograms are shown below. The most obvious difference is that a "family" must have at least two people. Otherwise, the family-size distribution has slightly larger probabilities for 2, 3, or 4, while for large family/household sizes, the differences between the distributions are small.





- **4.53.** (a) See also the solution to Exercise 4.22. If we view this time as being measured to any degree of accuracy, it is continuous; if it is rounded, it is discrete. (b) A count like this must be a whole number, so it is discrete. (c) Incomes—whether given in dollars and cents, or rounded to the nearest dollar—are discrete. (However, it is often useful to treat such variables as continuous.)
- **4.54.** (a) 0.8507 + 0.1448 + 0.0045 = 1. (b) Histogram on the right. (The third bar is so short that it blends in with the horizontal axis.) (c) P(at least one ace) = 0.1493, which can be computed either as 0.1448 + 0.0045 or 1 0.8507.

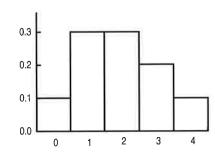


4.55. (a) Histogram on the right. (b) "At least one nonword error" is the event " $X \ge 1$ " (or "X > 0"). $P(X \ge 1) = 1 - P(X = 0) = 0.9$. (c) " $X \le 2$ " is "no more than two nonword errors," or "fewer than three nonword errors."

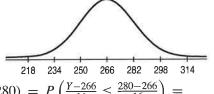
$$P(X \le 2) = 0.7 = P(X = 0) + P(X = 1) + P(X = 2)$$

= 0.1 + 0.3 + 0.3

$$P(X < 2) = 0.4 = P(X = 0) + P(X = 1) = 0.1 + 0.3$$



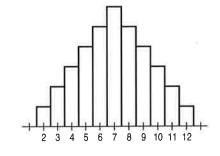
4.56. (a) Curve on the right. A good procedure is to draw the curve first, locate the points where the curvature changes, then mark the horizontal axis. Students may at first make mistakes like drawing a half-circle instead of the correct "bell-shaped" curve or being careless about



locating the standard deviation. (b) About 0.81: $P(Y \le 280) = P\left(\frac{Y-266}{16} \le \frac{280-266}{16}\right) = P(Z \le 0.875)$. Software gives 0.8092; Table A gives 0.8078 for 0.87 and 0.8106 for 0.88 (so the average is again 0.8092).

- **4.57.** (a) The pairs are given below. We must assume that we can distinguish between, for example, "(1,2)" and "(2,1)"; otherwise, the outcomes are not equally likely.
 - (b) Each pair has probability 1/36. (c) The value of X is given below each pair. For the distribution (given below), we see (for example) that there are four pairs that add to 5, so $P(X=5)=\frac{4}{36}$. Histogram below, right. (d) $P(7 \text{ or } 11)=\frac{6}{36}+\frac{2}{36}=\frac{8}{36}=\frac{2}{9}$.
 - (e) $P(\text{not } 7) = 1 \frac{6}{36} = \frac{5}{6}$.

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	3	4	5	- 6	7
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	4	5	6	7	- 8
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	5	6	7	8	9
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	6	7	8	9	10
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	7	8	9	.10	11
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)
7	8	9	10	11	12



Value of X	2	3	4	5	6	7	8	9	10	11	12
Probability	<u>1</u> 36	2 36	3 36	4 36	<u>5</u> 36	6 36	$\frac{5}{36}$	4 36	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

4.58. The possible values of Y are 1, 2, 3, ..., 12, each with probability 1/12. Aside from drawing a diagram showing all the possible combinations, one can reason that the first (regular) die is equally likely to show any number from 1 through 6. Half of the time, the second roll shows 0, and the rest of the time it shows 6. Each possible outcome therefore has probability $\frac{1}{6} \cdot \frac{1}{2}$.

4.59. The table on the right shows the additional columns to add to the table given in the solution to Exercise 4.57. There are 48 possible (equally-likely) combinations.

Value of X	2	3	4	5	6	7	8	9	10	11	12	13	14
Probability	1 48	2 48	3 48	4 48	<u>5</u> 48	6 48	6 48	<u>6</u> 48	<u>5</u> 48	$\frac{4}{48}$	$\frac{3}{48}$	$\frac{2}{48}$	$\frac{1}{48}$

(1,7)	(1,8)
8	9
(2,7)	(2,8)
9	10
(3,7)	(3,8)
10	11
(4,7)	(4,8)
11	12
(5,7)	(5,8)
12	13
(6,7)	(6,8)
13	14

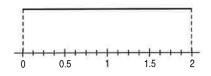
4.60. (a) W can be 0, 1, 2, or 3. (b) See the top two lines of the table below. (c) The distribution is given in the bottom two lines of the table. For example, $P(W = 0) = (0.73)(0.73)(0.73) \doteq 0.3890$, and in the same way, $P(W = 3) = 0.27^3 \doteq 0.1597$. For P(W = 1), note that each of the three arrangements that give W = 1 have probability (0.73)(0.73)(0.27) = 0.143883, so $P(W = 1) = 3(0.143883) \doteq 0.4316$. Similarly, $P(W = 2) = 3(0.73)(0.27)(0.27) \doteq 0.1597$.

Arrangement	DDD	DDF	DFD	FDD	FFD	FDF	DFF	FFF
Probability	0.3890	0.1439	0.1439	0.1439	0.0532	0.0532	0.0532	0.0197
Value of W	0		1			2		3
Probability	0.3890		0.4316			0.1597		0.0197

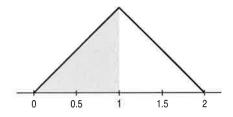
4.61. (a) P(X < 0.6) = 0.6. (b) $P(X \le 0.6) = 0.6$. (c) For continuous random variables, "equal to" has no effect on the probability; that is, P(X = c) = 0 for any value of c.

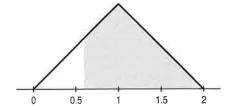
4.62. (a) $P(X \ge 0.30) = 0.7$. (b) P(X = 0.30) = 0. (c) P(0.30 < X < 1.30) = P(0.30 < X < 1) = 0.7. (d) $P(0.20 \le X \le 0.25 \text{ or } 0.7 \le X \le 0.9) = 0.05 + 0.2 = 0.25$. (e) $P(\text{not } [0.4 \le X \le 0.7]) = 1 - P(0.4 \le X \le 0.7) = 1 - 0.3 = 0.7$.

4.63. (a) The height should be $\frac{1}{2}$ since the area under the curve must be 1. The density curve is at the right. (b) $P(Y \le 1.6) = \frac{1.6}{2} = 0.8$. (c) $P(0.5 < Y < 1.7) = \frac{1.2}{2} = 0.6$. (d) $P(Y \ge 0.95) = \frac{1.05}{2} = 0.525$.



4.64. (a) The area of a triangle is $\frac{1}{2}bh = \frac{1}{2}(2)(1) = 1$. (b) P(Y < 1) = 0.5. (c) P(Y > 0.6) = 0.82; the easiest way to compute this is to note that the *unshaded* area is a triangle with area $\frac{1}{2}(0.6)(0.6) = 0.18$.





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4.65. $P(8 \le \bar{x} \le 10) = P\left(\frac{8-9}{0.0724} \le \frac{\bar{x}-9}{0.0724} \le \frac{10-9}{0.0724}\right) = P(-13.8 \le Z \le 13.8)$. This probability is essentially 1; \bar{x} will almost certainly estimate μ within ± 1 (in fact, it will almost certainly be much closer than this).

- **4.66.** (a) $P(0.52 \le \hat{p} \le 0.60) = P\left(\frac{0.52 0.56}{0.019} \le \frac{\hat{p} 0.56}{0.019} \le \frac{0.60 0.56}{0.019}\right) = P(-2.11 \le Z \le 2.11) = 0.9826 0.0174 = 0.9652$. (b) $P(\hat{p} \ge 0.72) = P\left(\frac{\hat{p} 0.56}{0.019} \ge \frac{0.72 0.56}{0.019}\right) = P(Z \ge 8.42)$; this is basically 0.
- **4.67.** The possible values of X are \$0 and \$1, each with probability 0.5 (because the coin is fair). The mean is $\$0\left(\frac{1}{2}\right) + \$1\left(\frac{1}{2}\right) = \0.50 .
- **4.69.** If Y = 15 + 8X, then $\mu_Y = 15 + 8\mu_X = 15 + 8(10) = 95$.
- **4.70.** If W = 0.5U + 0.5V, then $\mu_W = 0.5\mu_U + 0.5\mu_V = 0.5(20) + 0.5(20) = 20$.
- **4.71.** First we note that $\mu_X = 0(0.5) + 2(0.5) = 1$, so $\sigma_X^2 = (0-1)^2(0.5) + (2-1)^2(0.5) = 1$ and $\sigma_X = \sqrt{\sigma_X^2} = 1$.
- **4.72.** (a) Each toss of the coin is independent (that is, coins have no memory). (b) The variance is multiplied by $10^2 = 100$. (The mean and *standard deviation* are multiplied by 10.) (c) The correlation does not affect the mean of a sum (although it does affect the variance and standard deviation).
- **4.73.** The mean is

$$\mu_X = (0)(0.3) + (1)(0.1) + (2)(0.1) + (3)(0.2) + (4)(0.1) + (5)(0.2) = 2.3$$
 servings.

The variance is

$$\sigma_X^2 = (0 - 2.3)^2 (0.3) + (1 - 2.3)^2 (0.1) + (2 - 2.3)^2 (0.1) + (3 - 2.3)^2 (0.2) + (4 - 2.3)^2 (0.1) + (5 - 2.3)^2 (0.2) = 3.61,$$

so the standard deviation is $\sigma_X = \sqrt{3.61} = 1.9$ servings.

- **4.74.** The mean number of aces is $\mu_X = (0)(0.8507) + (1)(0.1448) + (2)(0.0045) = 0.1538$. **Note:** The exact value of the mean is 2/13, because 1/13 of the cards are aces, and two cards have been dealt to us.
- **4.75.** The average grade is $\mu = (0)(0.05) + (1)(0.04) + (2)(0.20) + (3)(0.40) + (4)(0.31) = 2.88$.
- 4.76. The means are

$$(0)(0.1) + (1)(0.3) + (2)(0.3) + (3)(0.2) + (4)(0.1) = 1.9$$
 nonword errors and $(0)(0.4) + (1)(0.3) + (2)(0.2) + (3)(0.1) = 1$ word error