Problems for Section 4.4

Homework # 14

1. Figure 4.48 shows cost and revenue. For what production levels is the profit function positive? Negative? Estimate the production at which profit is maximized.

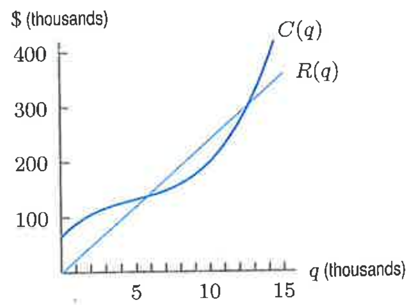

Figure 4.48

3. Table 4.2 shows cost, $C(q)$, and revenue, $R(q)$.

   (a) At approximately what production level, q, is profit maximized? Explain your reasoning.

   (b) What is the price of the product?

   (c) What are the fixed costs?

Table 4.2

q	0	500	1000	1500	2000	2500	3000
$R(q)$	0	1500	3000	4500	6000	7500	9000
$C(q)$	3000	3800	4200	4500	4800	5500	7400

5. Let $C(q)$ represent the cost, $R(q)$ the revenue, and $\pi(q)$ the total profit, in dollars, of producing q items.

   (a) If $C'(50) = 75$ and $R'(50) = 84$, approximately how much profit is earned by the 51^{st} item?

   (b) If $C'(90) = 71$ and $R'(90) = 68$, approximately how much profit is earned by the 91^{st} item?

   (c) If $\pi(q)$ is a maximum when $q = 78$, how do you think $C'(78)$ and $R'(78)$ compare? Explain.

7. Table 4.3 shows marginal cost, MC, and marginal revenue, MR.

   (a) Use the marginal cost and marginal revenue at a production of $q = 5000$ to determine whether production should be increased or decreased from 5000.

   (b) Estimate the production level that maximizes profit.

Table 4.3

q	5000	6000	7000	8000	9000	10000
MR	60	58	56	55	54	53
MC	48	52	54	55	58	63

9. A company estimates that the total revenue, R, in dollars, received from the sale of q items is $R = \ln(1 + 1000q^2)$. Calculate and interpret the marginal revenue if $q = 10$.

11. Figure 4.52 shows graphs of marginal cost and marginal revenue. Estimate the production levels that could maximize profit. Explain your reasoning.

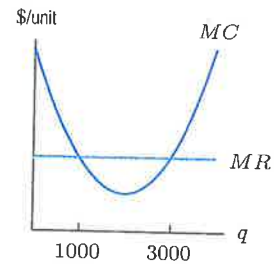

Figure 4.52

13. A manufacturing process has marginal costs given in the table; the item sells for $30 per unit. At how many quantities, q, does the profit appear to be a maximum? In what intervals do these quantities appear to lie?

q	0	10	20	30	40	50	60
MC ($/unit)	34	23	18	19	26	39	58

15. Cost and revenue functions are given in Figure 4.53.

    (a) At a production level of $q = 3000$, is marginal cost or marginal revenue greater? Explain what this tells you about whether production should be increased or decreased.

    (b) Answer the same questions for $q = 5000$.

17. Revenue is given by $R(q) = 450q$ and cost is given by $C(q) = 10{,}000 + 3q^2$. At what quantity is profit maximized? What is the total profit at this production level?

19. Revenue and cost functions for a company are given in Figure 4.54.

    (a) Estimate the marginal cost at $q = 400$.

    (b) Should the company produce the 500^{th} item? Why?

    (c) Estimate the quantity which maximizes profit.

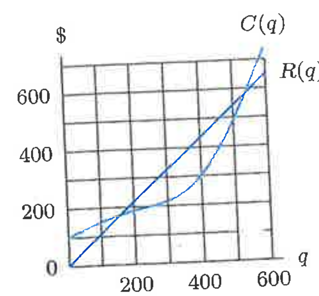

Figure 4.54

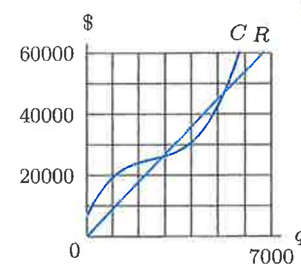

Figure 4.53

21. The demand for tickets to an amusement park is given by $p = 70 - 0.02q$, where p is the price of a ticket in dollars and q is the number of people attending at that price.

    (a) What price generates an attendance of 3000 people? What is the total revenue at that price? What is the total revenue if the price is $20?

    (b) Write the revenue function as a function of attendance, q, at the amusement park.

    (c) What attendance maximizes revenue?

    (d) What price should be charged to maximize revenue?

    (e) What is the maximum revenue? Can we determine the corresponding profit?

Solutions

Section 4.4

1 $5.5 < q < 12.5$ positive;
$0 < q < 5.5$ and $q > 12.5$ negative;
Maximum at $q \approx 9.5$

3 (a) $q = 2500$
(b) \$3 per unit
(c) \$3000

5 (a) \$9
(b) $-\$3$
(c) $C'(78) = R'(78)$

7 (a) Increase production
(b) $q = 8000$

9 \$0.20/item

11 $q = 0$ or $q = 3000$

13 One; between 40 and 50

15 (a) MR; increase production
(b) MC; decrease production

17 Global maximum of \$6875 at $q = 75$

19 (a) Approximately \$1
(b) No
(c) About 400 items

21 (a) \$10; \$30,000; \$50,000
(b) $R(q) = 70q - 0.02q^2$
(c) 1750
(d) \$35
(e) \$61,250

23 \$14.

25 (a) $10{,}000 + 2q$
(b) $q = 37{,}820 - 5544p$
(c) $\pi = -0.00018q^2 + 4.822q - 10{,}000$
(d) 13,394 items, \$22,294

27 Maximum revenue = \$27,225
Minimum = \$0

29 (a) q/r months
(b) $(ra/q) + rb$ dollars
(c) $C = (ra/q) + rb + kq/2$ dollars
(d) $q = \sqrt{2ra/k}$

31 $L = [\beta pcK^{\alpha}/w]^{1/(1-\beta)}$