Calc Math 211 2015S W13 Tues
Problem 6 from §6.3

A bond is guaranteed to pay $100+10t$ dollars per year for 10 years where t is in years from the present.

Find the present value of this income stream given an interest rate of 5%, compounded continuously.

If you are offered this bond or a lump sum payment of <u>fill in blank</u> which should you take?

$$P = \int_0^m S(t)\, e^{-rt}\, dt$$

$$\int_0^{10} (100+10t)\, e^{(-0.05)t}\, dt$$

How do you take this integral

There is a trick that I'm going to show you, It comes from the product Rule.

$$d(uv) = u\,dv + v\,du$$

$$u\,dv = d(uv) - v\,du$$

$$\int u\,dv = uv - \int v\,du$$

$u = 100 + 10t$ $\qquad$ $dv = e^{-0.05t}\,dt$

$du = 10\,dt$ $\qquad$ $v = \frac{1}{-0.05} e^{-0.05t}$

Integral $\int u\,dv$ $= (100+10t)\frac{1}{-0.05} e^{-0.05t}\Big|_0^{10}$

$$-\int_0^{10} \frac{1}{-0.05} e^{-0.05t} \cdot 10\,dt$$

$$\frac{100+10\cdot 10}{-0.05} e^{-0.05\cdot 10} - \frac{100}{-0.05}e^{0} + \frac{-10}{(0.05)^2} e^{-0.05t}\Big|_0^{10}$$

$$\frac{-1}{0.05}\left(200 e^{-.5} - 100\right) + \frac{10}{(0.05)^2}\left(1 - e^{-0.05\cdot 10}\right)$$

$$\boxed{1483.63}$$

If someone offers you a lump sum of more than 1483.63 you should take it. Otherwise take the bond.

This was more complicated than I'd expect on an exam.

Here are some simpler examples;

$\boxed{\text{Eg2}}$ $\int x e^{3x}\,dx$

$u = x \qquad dv = e^{3x}\,dx$

$du = dx \qquad v = \frac{1}{3}e^{3x}$

Integral $= \frac{x}{3}e^{3x} - \int \frac{1}{3}e^{3x}\,dx$

$= \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$

$\boxed{\text{Eg3}}$ $\int_2^3 \ln x\,dx$

$u = \ln(x) \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

Integral $= x\ln x\Big|_2^3 - \int_2^3 \frac{x}{x}\,dx$

$= x\ln x - x\Big|_2^3$

$= 3\ln 3 - 3 - 2\ln 2 + 2$

$= 3\ln 3 - 2\ln 2 - 1$

Eg 4 $\int x^6 \ln x \, dx$

$u = \ln x \qquad dv = x^6 \, dx$

$du = \frac{1}{x} dx \qquad v = \frac{1}{7} x^7$

Integral $= \frac{1}{7} x^7 \ln x - \int \frac{1}{7} x^7 \cdot \frac{1}{x} \, dx$

$= \frac{1}{7} x^7 \ln x - \frac{1}{7} \int x^6 \, dx$

$= \frac{1}{7} x^7 \ln x - \frac{1}{49} x^7 + C$