Math 211
Summer 2014
Final Exam—2nd Practice
6/26/14
Time Limit: 120 Minutes

Name (Print): Answers

This exam contains 9 pages (including this cover page) and 11 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. Graphing calculators should not be needed, but they can be used to check your work. If you use a graphing calculator to find an answer you must write the steps needed to find the answer, without the calculator.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	5	
11	5	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$
\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= 7\cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\
\int x^n\,dx &= \frac{x^{n+1}}{n+1}+C, \quad n\neq -1 \\
\int \frac{1}{x}\,dx &= \ln(x)+C \\
\int e^{kx}\,dx &= \frac{1}{k}e^{kx}+C \\
\int (f(x)+g(x))\,dx &= \int f(x)\,dx+\int g(x)\,dx \\
\int cf(x)\,dx &= c\int f(x)\,dx
\end{aligned}
$$

1. (10 points) The elasticity of a good is $E = 0.75$. What is the effect of a 2% increase of price on the quantity demanded.

$$E = \left| \frac{\Delta q / q}{\Delta p / p} \right| = 0.75 \qquad \frac{\Delta p}{p} = 2\%$$

$$\left| \frac{\Delta q}{q} \right| = 2\% \cdot 0.75$$
$$= 1.5\%$$

A 2% increase in price will lead to a 1.5% decrease in demand.

2. (10 points) The demand for a product is given by $q = 150 - 3p^2$. Find the elasticity of demand when the price is \$5.

$$p = 5 \qquad q = 150 - 3 \cdot 5^2 = 150 - 75 = 75$$

$$E = \left| \frac{p}{q} \frac{dq}{dp} \right| \qquad q = 150 - 3p^2$$

$$\frac{dq}{dp} = -6p = -30$$

$$E = \left| \frac{5}{75} \cdot (-30) \right| = 2$$

Elasticity of demand is 2.

3. (10 points) Find the present and future value of an \$12,000 per year income stream over 10 years, with a discount rate of 5%.

$$PV = \int_0^M S(t)e^{-rt}\,dt = \int_0^{10} 12000\, e^{-0.05t}\,dt$$

$$= \frac{12000}{-0.05}\, e^{-0.05t}\Big|_0^{10}$$

$$= \frac{12000}{0.05}\left(1 - e^{-0.05\cdot 10}\right)$$

Present Value \$94432.64

Future Value $FV = Pe^{rm} = (94,432.64)\, e^{0.05\cdot 10}$

\$155,639

4. (10 points) Find the average value of the function $f(x) = \frac{1}{x}$ within the interval $[1, 5]$.

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{5-1}\int_1^5 \frac{1}{x}\,dx$$

$$= \frac{1}{4}\ln|x|\,\Big|_1^5$$

$$= \frac{1}{4}\left(\ln(5) - \ln(1)\right)$$

$$= \frac{\ln(5)}{4} = 0.402$$

5. (10 points) The marginal cost function of a product is $C'(q) = q^2 - 50q + 700$. The fixed costs are \$500. Find the total cost to produce 50 items.

$$TC(q) = FC + \int_0^b MC(q)\,dq$$

$$TC(50) = 500 + \int_0^{50} (q^2 - 50q + 700)\,dq$$

$$= 500 + \frac{q^3}{3} - 25q^2 + 700q \Big|_0^{50}$$

$$= 500 + \frac{50^3}{3} - 25 \cdot 50^2 + 700 \cdot 50 - 0$$

$$= 14,666.67$$

6. (10 points) The relative rate of growth of a population, between time t=2, and time t=5, is given by $r(t) = q^2 + q + 1$. Relate the population at time 5, to the population at time 2.

$$p(b) = p(a)\,e^{I}$$

$$I = \int_2^5 (q^2 + q + 1)\,dq$$

$$= \frac{q^3}{3} + \frac{q^2}{2} + q$$

$$= \frac{5^3}{3} + \frac{5^2}{2} + 5 - \frac{3^3}{3} - \frac{3^2}{2} - 3$$

$$= 42.6667$$

The population at time 5 is $e^{42.6667}$ times the population at time 2.

7. (10 points) A bus company has fixed costs. For \$3 per ride the company attracts 600 passengers. With each additional \$0.05 the company loses 15 passengers (assume a linear demand eequation). How should the bus company set its price to maximize profits?

Fixed Cost: $C' = 0$

$\pi = R - C$

$\pi' = R' - 0$

Maximizing Revenue maximizes profit

$R = p \cdot q$ How do we get q?

Demand Function

$q = mp + b$ $m = \frac{\Delta q}{\Delta p} = \frac{-15}{0.05} = -300$

$600 = (-300)(3) + b$ $b = 600 + 900 = 1500$

Demand $q = -300p + 1500$

Revenue $= p \cdot q = -300p^2 + 1500p$

Critical points of Revenue/Profit $R'(p) = 0$

$-600p + 1500 = 0$ $p = 2.5$

maximum because $R''(p) = -600 < 0$

Company should lower price by 50¢ to \$2.50 to maximize profits.

8. (10 points) The local linear approximation is given by the formula

$$f(x) = f(a) + f'(a)(x - a).$$

Suppose the value of the function is 5, and its derivative is 6, both at $x = 3$. Approximate the function at $x = 5$.

$f(3) = 5$

$f'(3) = 6$

want $f(5)$ plug in $a=3$ $x=5$

$f(5) = f(3) + f'(3)(5-3)$

$f(5) = 5 + 6 \cdot 2$

$= 17$

9. (10 points) Consider the function $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + 10$. On what intervals is f increasing? On what intervals is f decreasing? Where are the critical points of f (if any)? Where are the local minima of f (if any)? Where are the local maxima of f (if any)?

f is increasing where $f'(x)$ is positive
decreasing where $f'(x)$ is negative
f has critical points where $f(x)=0$

$$f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + 10$$

$$f'(x) = x^2 - 5x + 4 = (x-4)(x-1)$$

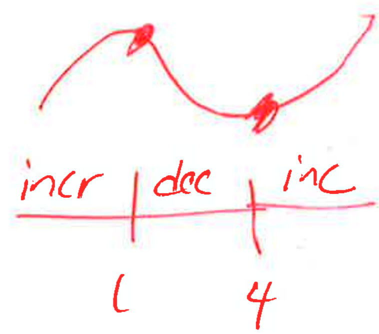
$f'(x) = 0$ if and only if

$x = 4$ or $x = 1$ only at these points can f change from increasing to decreasing or vice versa

$f'(2) = 4 - 10 + 4 = -2 < 0$

$f'(0) = 4 > 0$

$f'(5) = 25 - 25 + 4 = 4 > 0$



f is increasing for $x \leq 1$ or $x \geq 4$
f is decreasing for $1 < x < 4$
f has critical points at 1 and 4; Finally f has local max at 1 and local min at 4

10. (5 points) Find the following integral by substitution:

$$I = \frac{1}{10}\int \frac{10q}{5q^2+8}dq$$

$$u = 5q^2+8$$
$$du = 10q\,dq$$

$$I = \frac{1}{10}\int \frac{du}{u} = \frac{1}{10}\ln|u| + C$$

$$= \frac{1}{10}\ln(5q^2+8) + C$$

11. (5 points) Find the following integral by integration by parts:

$$I = \int ln(t)dt$$

$$u = \ln(t) \qquad dv = dt$$
$$du = \frac{1}{t}dt \qquad v = t$$

$$I = t\ln(t) - \int \frac{\cancel{t}}{\cancel{t}}\,dt$$

$$= t\ln t - t + C$$