Math 211 - 2015S - W7 - ~~Wed~~ Friday Pg 1

Average Cost

If the cost of producing a quantity q of a good is $C(q)$ then the average cost of producing a quantity q is

$$a(q) = \frac{C(q)}{q}$$

If you sell everything you make and if average cost is less than price you make money

Otherwise if $a(q) > p$ lose money

$p > a(q)$ make money

$$p > \frac{c(q)}{q}$$

$$pq > C(q)$$

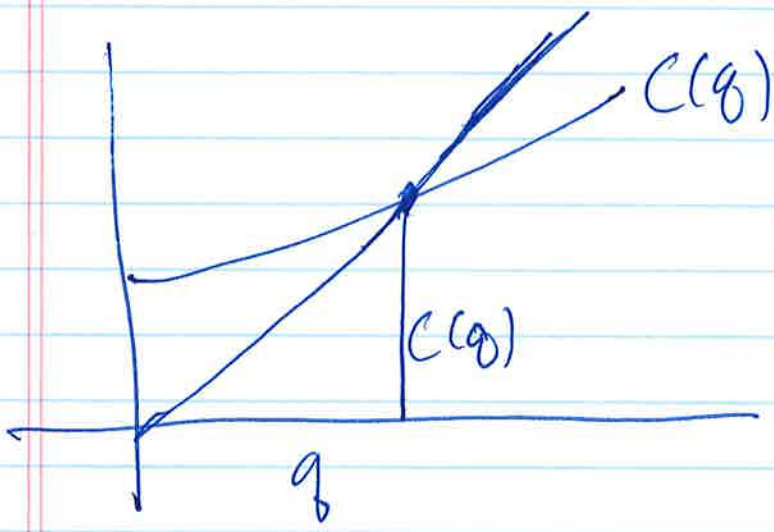
$$R(q) - C(q) > 0$$

$$\Pi(q) > 0$$

The other things average cost tells you is how likely other firms are to enter market

If average cost is low firms are more likely to enter market.

Plotting average cost



What line has slope a(q)?

rise/run $= \dfrac{C(q)}{q}$

Line connecting origin to point on cost function

Eg if $C(q) = 1000 + 20q$

(a) Find MC of tenth item
(b) Find AC of tenth item

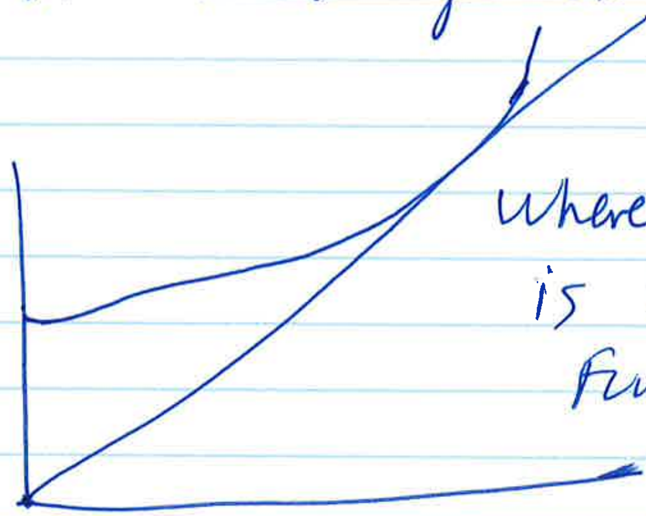
(a) $MC = 20$

(b) $AC = \frac{C(10)}{10} = \frac{1200}{10} = 120$

Units of both are \$/unit

eg \$/lbs if unit is lbs of butter

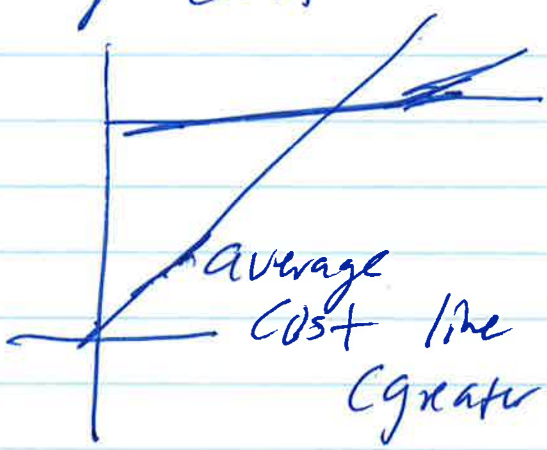
Where is average cost minimized



Where line from origin is tangent to Cost function

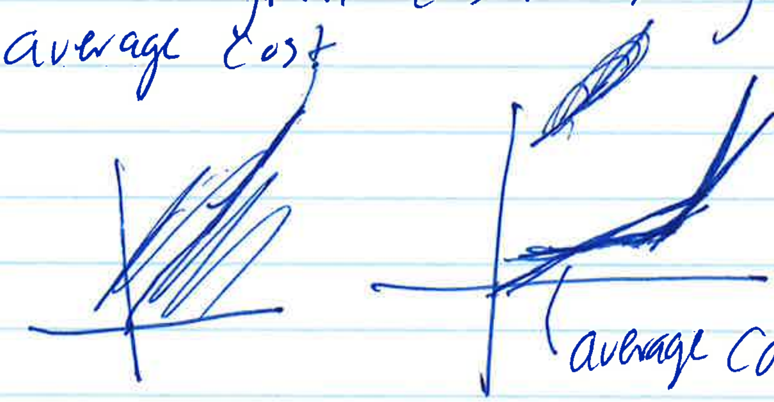
Relationship between average cost and marginal cost

If marginal cost is less than average cost



Then increasing production (traversing cost curve to right) decreases average cost

If marginal cost is greater than average cost



then increasing production increases average costs

Finally: Marginal cost equals average cost at critical points of average cost.

Elasticity of demand

- measures sensitivity of demand to changes in price

$$E \approx \left| \frac{\text{percent change in demand}}{\text{percent change in price}} \right|$$

approximate because real definition involves derivatives

If a 1% increase in price leads to a 1% decrease in demand $E \approx 1$

changes happen in opposite direction hence the absolute value one sign + one sign −

If a 1% increase in price leads to a 2% ~~de~~ decrease in demand $E \approx 2$

If a 2% change in price leads to a 1% change in demand $E \approx 1/2$

$E > 1$ called elastic

$E < 1$ called inelastic

Percent change in demand (relative change)

$$\frac{\Delta q}{q}$$

Percent change in price $\Delta p/p$

$$E \approx \left| \frac{\frac{\Delta q}{q}}{\Delta p/p} \right| = \left| \frac{\Delta q}{q} \cdot \frac{p}{\Delta p} \right|$$

$$= \left| \frac{p}{q} \frac{\Delta q}{\Delta p} \right|$$

difference quotient

Replace $\frac{\Delta q}{\Delta p}$ with $\frac{dq}{dp}$ and get

true definition of E not approximation

$$E = \left| \frac{p}{q} \frac{dq}{dp} \right|$$

Another formula

$$E \approx \left| \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \right|$$

Take away absolute value
(put in minus sign)

$$E \approx -\frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}}$$

or

$$\boxed{\frac{\Delta q}{q} \approx -E\,\frac{\Delta p}{p}}$$

relative change in demand

relative change in price

Eg: Find elasticity of demand at price $p=10$ if demand curve is given by

$$q = 1000 - 2p^2$$

$$p = 10 \Rightarrow q = 1000 - 200 = 800$$

$$\frac{dq}{dp} = -4p = -40$$

$$E = \left| \frac{p}{q} \frac{dq}{dp} \right| = \left| \frac{10}{800}(-40) \right|$$

$$= \frac{1}{2}$$

Revenue and Elasticity of Demand

Increase in price leads to decrease in demand (usually/always as far as we are concerned)

Revenue may increase or decrease

If $E<1$ (inelastic)

Revenue increases when price increases

If $E>1$ (elastic)

Revenue decreases when price increases

Petroleum companies make more money when prices goes up (due to market forces, war in middle east, etc)

Apple farmers make less money when the price of their good goes up

When a product is replacable it is likely to be elastic (oranges instead of apples)
When irreplaceable (petroleum) it is likely to be inelastic

E = 1 occurs at critical points of revenue function as a function of price

Slope of revenue function can change sign only when E = 1

$$R' = \frac{dR}{dp} = \frac{d}{dp}(pq) = p\frac{dq}{dp} + q\frac{dp}{dp}$$

$$= p\frac{dq}{dp} + q$$

When $R' = 0$

$$p\frac{dq}{dp} + q = 0$$

$$p\frac{dq}{dp} = -q$$

$$\frac{p}{q}\frac{dq}{dp} = -1$$

$$E = \left|\frac{p}{q}\frac{dq}{dp}\right| = 1$$