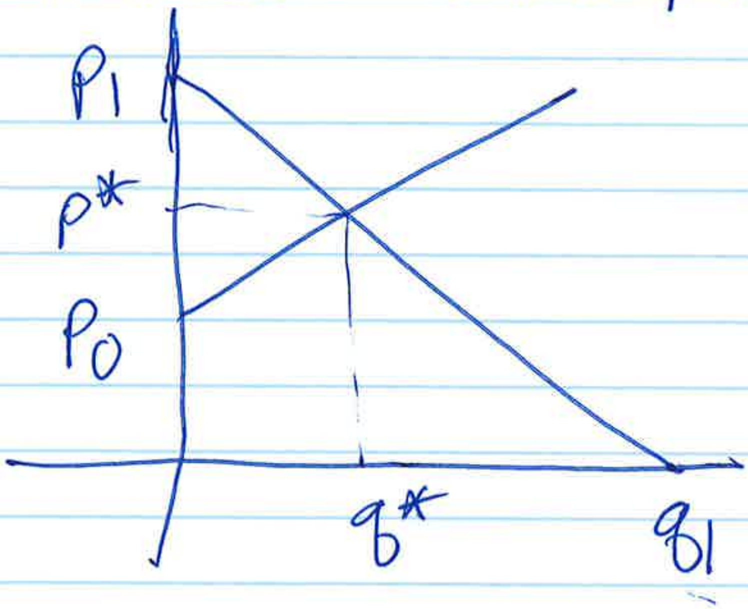
Math 211 - 2015S - W2 - Wed (Pg 1)

Review

Supply curve - relates price to quantity supplied
demand curve - relates price to quanity demanded



Eq $q = 3p - 50$ supply curve (pos slope)
$q = 100 - 2p$ demand curve (neg slope)

To find equilibrium price set q^*'s equal and solve for p^*

Taxation affect equilibrium

A tax on consumers ~~affects~~ increases the price that they pay ~~(increases it)~~

$$100-2p=q$$

becomes

$100-2(p+5)=q$ for a \$5 specific tax

$100-2(1.05)p=q$ for a 5% sales tax

in both cases ~~demand~~ supply curve remains the same

A tax on suppliers decreases the price they collect

On supply curve

$$q=3p-50$$

becomes

$q=3(p-5)-50$ specific tax

$q=3(\cancel{1.05}\ .95)p-50$ sales tax

etc.

A subsidy does reverse

- decreases price for consumers (demand curve)
- increases price for suppliers (supply curve)

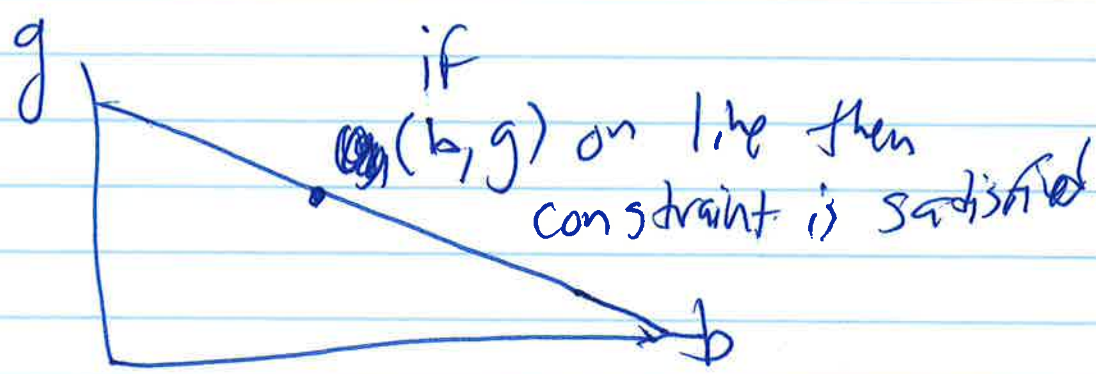
one or the other depending on how ~~tax is levied~~, subsidy is given.

Budget constraint

A linear equation relating e.g. guns and butter

$$400g + 2000b = 12000$$

- 400 ↑ price of guns
- g: # guns
- 2000 ↑ price of butter
- b: # butter
- 12000 ← total expendature



Exponential Functions

Examples $F(x) = 2^x$

$F(x) = 3e^{4x}$ ← Variable in exponent

constant base

E.g. population of Nevada

$P = 2.02(1.036)^t$ (millions) $\quad P = P_0 a^t$

population when $t=0$ — growth factor

When $t=1$ $\quad P = (2.02)(1.036)$

$t=2$ $\quad P = (2.02)(1.036)^2$

$= (2.02)(1.036)(1.036)$

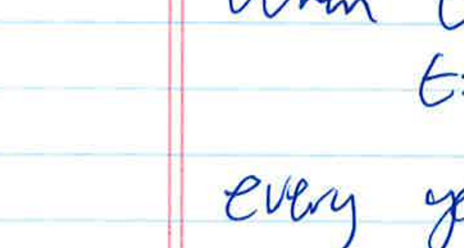
every year multiply by 1.036

Percentage increase

$r = a - 1$ in percent

$1 - 1.036 = .036$

3.6% increase per year



Elimination of drug from body

Exponential decay

(growth factor < 1

(percentage increase < 0

$$P = P_0 a^t$$

$$P = 250(.6)^t$$

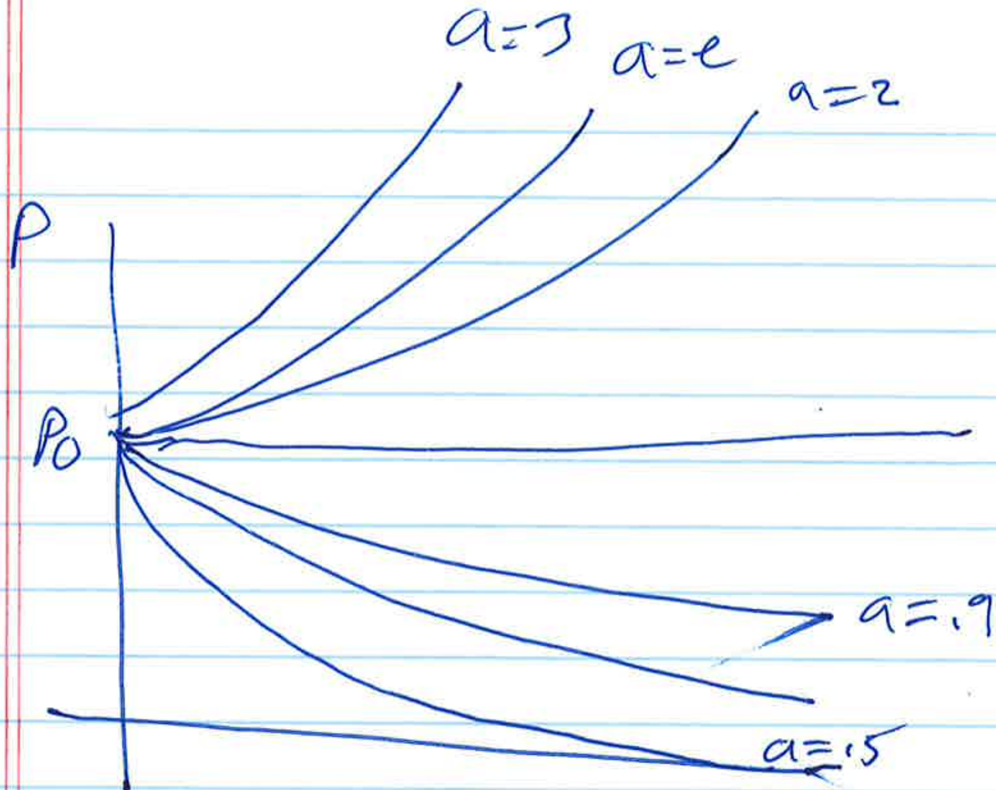
$$a = .6$$

same formula $\quad r = a - 1 = -.4$ or -40%

Comparison between linear and exponential growth

linear - constant rate of change
(e.g. increase y by 6 every time increase x by 1 ; slope $= \frac{rise}{run} = \frac{6}{1}$)

exponential - constant relative rate of change
(e.g. increase y by 6% every time increase x by 1.



e is a special case

It makes calculus formulas come out nicely which is why it is used often in calculus

Natural logarithm

$\ln x = c$ means $e^c = x$

Properties

$\ln(AB) = \ln(A) + \ln(B)$
$\ln(A/B) = \ln(A) - \ln(B)$
$\ln(A^p) = p \ln(A)$

$e^{A+B} = e^A e^B$
$e^{A-B} = e^A / e^B$
$(e^n)^m = e^{nm}$

$\ln(e^x) = x$

$e^{\ln(x)} = x$

$\ln(1) = 0$ since $e^0 = 1$
$\ln(e) = 1$ since $e^1 = e$

$3^t = 10$ solve

$\ln(3^t) = \ln(10)$

$t\ln(3) = \ln(10) \Rightarrow t = \frac{\ln(10)}{\ln(3)} = 2.096$

$12 = 5e^{3t}$ solve

$\frac{12}{5} = e^{3t} \Rightarrow \ln\left(\frac{12}{5}\right) = \ln(e^{3t}) = 3t$

$t = \ln\left(\frac{12}{5}\right)/3 = 0.2918$

$P = P_0 a^t$ or $P = P_0 e^{kt}$

Can be written in either form

Write

$a = e^k$ or $k = \ln(a)$

$P = P_0 (e^k)^t = P_0 e^{kt}$