

# Homework #10

## Math 211

### Problems for Section 3.1

For Problems 1–36, find the derivative. Assume  $a, b, c, k$  are constants.

1.  $y = 5$

3.  $y = x^{12}$

5.  $y = x^{4/3}$

7.  $y = 3t^4 - 2t^2$

9.  $f(x) = \frac{1}{x^4}$

11.  $y = x^2 + 5x + 9$

13.  $y = 3x^2 + 7x - 9$

15.  $y = 4.2q^2 - 0.5q + 11.27$

17.  $g(t) = \frac{1}{t^5}$

19.  $y = \frac{1}{r^{7/2}}$

21.  $h(\theta) = \frac{1}{\sqrt[3]{\theta}}$

23.  $y = 3t^5 - 5\sqrt{t} + \frac{7}{t}$

25.  $y = 3t^2 + \frac{12}{\sqrt{t}} - \frac{1}{t^2}$

27.  $y = \sqrt{x}(x+1)$

29.  $f(x) = kx^2$

31.  $Q = aP^2 + bP^3$

33.  $P = a + b\sqrt{t}$

35.  $w = 3ab^2q$

### Problems for Section 3.1

41. The height of a sand dune (in centimeters) is represented by  $f(t) = 700 - 3t^2$ , where  $t$  is measured in years since 2005. Find  $f(5)$  and  $f'(5)$ . Using units, explain what each means in terms of the sand dune.

43. The quantity,  $Q$ , in tons, of material at a municipal waste site is a function of the number of years since 2000, with

$$Q = f(t) = 3t^2 + 100.$$

Find  $f(10)$ ,  $f'(10)$ , and the relative rate of change  $f'/f$  at  $t = 10$ . Interpret your answers in terms of waste.

45. If  $f(t) = 2t^3 - 4t^2 + 3t - 1$ , find  $f'(t)$  and  $f''(t)$ .

49. Find the equation of the line tangent to the graph of  $f$  at  $(1, 1)$ , where  $f$  is given by  $f(x) = 2x^3 - 2x^2 + 1$ .

55. The cost to produce  $q$  items is  $C(q) = 1000 + 2q^2$  dollars. Find the marginal cost of producing the 25<sup>th</sup> item. Interpret your answer in terms of costs.

63. If the demand curve is a line, we can write  $p = b + mq$ , where  $p$  is the price of the product,  $q$  is the quantity sold at that price, and  $b$  and  $m$  are constants.

- (a) Write the revenue as a function of quantity sold.  
 (b) Find the marginal revenue function.

### Problems for Section 3.2

Differentiate the functions in Problems 1–28. Assume that  $A$ ,  $B$ , and  $C$  are constants.

1.  $f(x) = 2e^x + x^2$

13.  $y = e^{-4t}$

3.  $y = 5t^2 + 4e^t$

15.  $P = 50e^{-0.6t}$

5.  $y = 2^x + \frac{2}{x^3}$

17.  $P(t) = 3000(1.02)^t$

7.  $f(x) = 2^x + 2 \cdot 3^x$

21.  $f(x) = Ae^x - Bx^2 + C$

9.  $y = 3x - 2 \cdot 4^x$

23.  $R = 3 \ln q$

11.  $f(t) = e^{3t}$

25.  $y = t^2 + 5 \ln t$

27.  $y = x^2 + 4x - 3 \ln x$

## Section 3.1

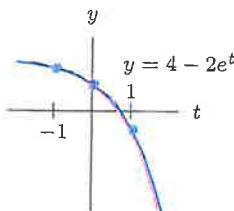
- 41 Height = 625 cm,  
Changing (eroding) at -30 cm/year  
 43  $f(10) = 400$  tons;  
 $f'(10) = 60$  tons per year;  
 Relative rate = 15% per year  
 45  $f'(t) = 6t^2 - 8t + 3$   
 $f''(t) = 12t - 8$   
 47 (a)  $f(100) = 11.11$  seconds  
 (b)  $f'(100) = 0.05555$  seconds per foot  
 49  $y = 2x - 1$   
 55 \$100  
 57 (a) 770 bushels per acre  
 (b) 40 bushels per acre per pound of fertilizer  
 (c) Use more fertilizer  
 59 (a)  $dC/dq = 0.24q^2 + 75$   
 (b)  $C(50) = \$14,750; C'(50) = \$675$  per item  
 61  $f'(x) = 3x^2 - 12x - 15$ ,  
 $x = -1$  and  $x = 5$

- 63 (a)  $R(q) = bq + mq^2$   
 (b)  $R'(q) = b + 2mq$

## Section 3.2

- 1 0  
 3  $12x^{11}$   
 5  $\frac{4}{3}x^{1/3}$   
 7  $12t^3 - 4t$   
 9  $-4x^{-5}$   
 11  $2x + 5$   
 13  $6x + 7$   
 15  $8.4q - 0.5$   
 17  $-5t^{-6}$   
 19  $-(7/2)r^{-9/2}$   
 21  $-(1/3)\theta^{-4/3}$   
 23  $15t^4 - \frac{5}{2}t^{-1/2} - 7t^{-2}$   
 25  $6t - 6/t^{3/2} + 2/t^3$   
 27  $(3/2)x^{1/2} + (1/2)x^{-1/2}$   
 29  $2kx$   
 31  $2aP + 3bP^2$   
 33  $b/(2\sqrt{t})$   
 35  $3ab^2$

- 1  $2e^x + 2x$   
 3  $10t + 4e^t$   
 5  $(\ln 2)2^x - 6x^{-4}$   
 7  $(\ln 2)2^x + 2(\ln 3)3^x$   
 9  $3 - 2(\ln 4)4^x$   
 11  $3e^{3t}$   
 13  $-4e^{-4t}$   
 15  $-30e^{-0.6t}$   
 17  $3000(\ln 1.02)(1.02)^t$   
 19  $Ce^t$   
 21  $Ae^x - 2Bx$   
 23  $3/q$   
 25  $2t + 5/t$   
 27  $2x + 4 - 3/x$   
 29  $f'(-1) \approx -0.736$   
 $f'(0) = -2$   
 $f'(1) \approx -5.437$



- 31  $y = -2t + 1$   
 33 (a) 13, 394 fish  
 (b) 8037 fish/month  
 35  $f(2) = 6065, f'(2) = -1516$   
 37  $f(5) = \$563.30;$   
 $f'(5) = \$70$  per week;  
 Relative rate = 12.4% per week  
 39 -444.3 people/year  
 41  $c = -1/\ln 2$   
 43  $C(50) \approx 1365, C'(50) \approx 18.27$   
 45 (a) 0.021 micrograms/year  
 (b) 779.4 years old in 1998  
 47 (a)  $P = 1.166(1.015)^t$   
 (b)  $\frac{dP}{dt} = 1.166(1.015)^t(\ln 1.015)$   
 $\frac{dP}{dt}|_{t=0} = 0.017$  billion people per year  
 $\frac{dP}{dt}|_{t=25} = 0.025$  billion people per year  
 49 (a)  $y = x - 1$   
 (b) 0.1; 1  
 (c) Yes