Math 211 2015S W4 - Wed

Review

See sheet (next page)

New

Chain Rule - Rule for differentiating composition of functions

If $y = f(z)$
and $z = g(t)$

Then the derivative of $y = f(g(t))$ is given by

$$\frac{dy}{dt} = \frac{dy}{dz}\frac{dz}{dt}$$ Leibniz Notation

$$\frac{d}{dt} f(g(t)) = f'(g(t))g'(t)$$ Newton notation

Example

Gas is consumed at a rate per mile $\frac{dG}{ds} = 0.05$ gallons/mi

Velocity is $\frac{ds}{dt} = 30$ mile/hr

Gas is consumed at a rate per hour $\frac{dG}{dt} = \frac{dG}{ds}\frac{ds}{dt}$

$= 0.05 \times 30$

$= 1.5$ gallons/hr

P92

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

Review

Do examples

$$
\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x)
\end{aligned}
$$

today →

Chain Rule

If you are given a formula that consists of a composite function, you should recognize it as such and then identify the
inner function
and outer function

Examples

a) $y = \ln(3t)$

inner $z = 3t$
outer $y = \ln(z)$

b) $P = e^{-0.03t}$

inner $z = 0.03t$
outer $p = e^{z}$

c) $w = 5(2r+3)^2$

inner $z = 2r+3$
outer $w = 5z^2$

Derivative

$\frac{dy}{dt} = \frac{dy}{dz}\frac{dz}{dt}$

$\frac{1}{z} \cdot 3 = \frac{1}{3t} \cdot 3$

$= \frac{1}{t}$

$\frac{dP}{dt} = \frac{dP}{dz}\frac{dz}{dt}$

$= e^{z}(-0.03)$

$(-0.03)e^{-0.03t}$

$\frac{dw}{dr} = \frac{dw}{dz}\frac{dz}{dr}$

$= 10z \cdot 2$

$= 10(2r+3) \cdot 2$

Most people prefer not to write down the inner and outer functions but to do it in their head using the following modified formula. If z is any differentiable function of t then

$$\frac{d}{dt}(z^n) = n z^{n-1}\left(\frac{dz}{dt}\right)$$

$$\frac{d}{dt}(e^z) = e^z \frac{dz}{dt}$$

$$\frac{d}{dt}(\ln z) = \frac{1}{z}\frac{dz}{dt}$$

Examples

Differentiate $\frac{d}{dt}(3t^3 - t)^5 = 5(3t^3 - t)(9t^2 - 1)$

$$\frac{d}{dq}\ln(q^2+1) = \frac{1}{q^2+1}\cdot 2q$$

$$\frac{d}{dx} e^{-x^2} = e^{-x^2}(-2x)$$

More Examples

$$\frac{d}{dx}(x^2+4)^3 = 3(x^2+4)^2(2x)$$

$$\frac{d}{dt}\, 5\ln(2t^2+3) = \frac{5}{2t^2+3}\cdot 4t$$

$$\frac{d}{dt}\left(\sqrt{1+2e^{5t}}\right) = \frac{d}{dt}\left(1+2e^{5t}\right)^{1/2}$$

$$= \frac{1}{2}\left(1+2e^{5t}\right)^{-1/2} 10e^{5t}$$

Relative Rate of Change

$$= \frac{F'(t)}{F(t)}$$

← equal

$$\underbrace{\frac{d}{dt}\left(\ln(F(t))\right)}_{\text{this is relative rate of change}} = \frac{1}{F(t)}\cdot F'(t)$$