Math 211-2015S W3-Tues Pg 1

Review

Financial Applications; Compound interest

P_0 is initial deposit
$P(t)$ is balance at time t

Compounded annually and Compounded continuously } equivalent but use different formulas

Annual: $P = P_0(1+r)^t$ where r is the annual interest rate

Compounded continuous:

$$P = P_0 e^{rt}$$ where r is the continuous interest rate

Can convert by changing r

$$(1 + r_{annual}) = e^{r_{continuous}}$$

Solve for one in terms of other.
Often annual is used if it is only calculated once per year, continuous if calculated continuously (although not necessarily the case).

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We also talked about

Composite functions $f(g(x))$ or $g(f(x))$

for two functions f and g

Power law

$Q = kx^p$ Q is a power function of X.

Proportional

$Q = kX$ Q is proportional to X

New

Before moving on to derivatives, I want to discuss one thing concerning financial applications.

<u>Present and Future Value</u> - we will revisit this topic after learning integration

The <u>future value</u> B of a payment P today is the amount to which P ~~would have~~ will grow by the future date if deposited in an interest bearing account today

$$B = P(1+r)^t \quad \text{annual compounding}$$

later we will deal exclusively with this formula →

$$B = Pe^{rt} \quad \text{continuous compounding}$$

B is the <u>future value</u>

P is the <u>present value</u>

The present value P (value today)

of a future payment B (payment promised on some date in future)

is the amount that would have to be deposited today to produce exactly B in account on the promised date in the future.

P is necessarily less than B

In fact it is the same equations

$$B = P(1+r)^t$$
$$B = Pe^{rt}$$

Except now we want to solve for P

$$P = B(1+r)^{-t} = B/(1+r)^t$$
$$P = B(e^{-rt}) = B/e^{rt}$$

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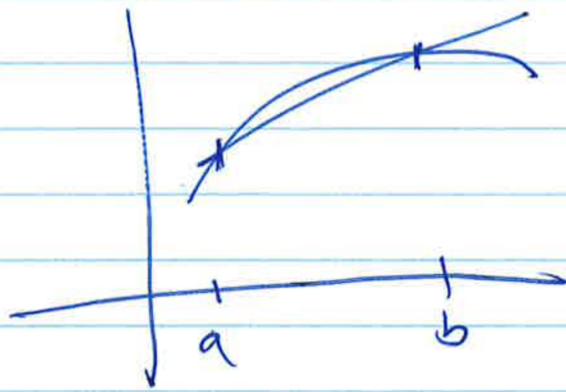
Chapter 2 Section 1

Remember <u>average rate of change</u> of a function between two points



The average rate of change of F is the value of the difference quotient

$$\frac{f(b)-F(a)}{b-a}$$

Which is the slope of the secant line connecting $(a, F(a))$ and $(b, F(b))$.

Now we are going to define the instantaneous rate of change of F at $\underline{\underline{a}}$ (one point not two)

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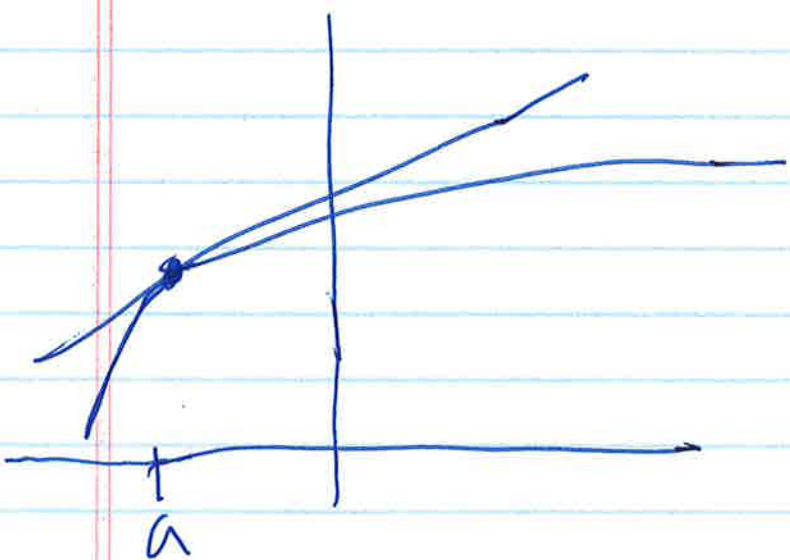
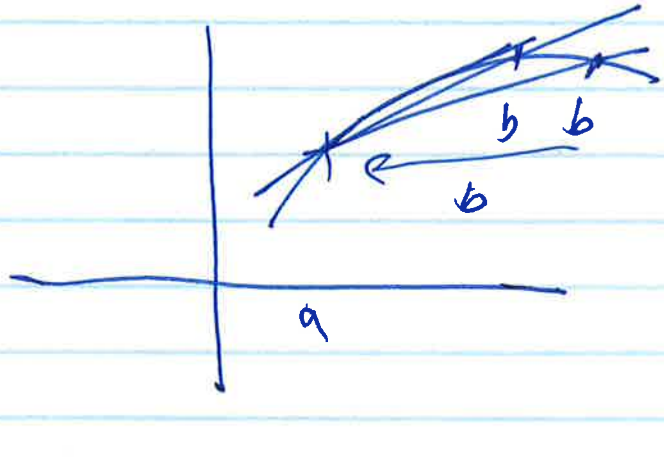
The instantaneous rate of change of f at a is the slope of the tangent line at a



this is the slope of the secant line as ~~[illegible]~~ b gets close to a



The instantaneous rate of change is also called the derivative of f at a.

There are a couple of notations for this quantity (IRofC of f at a) or derivative of f at a.

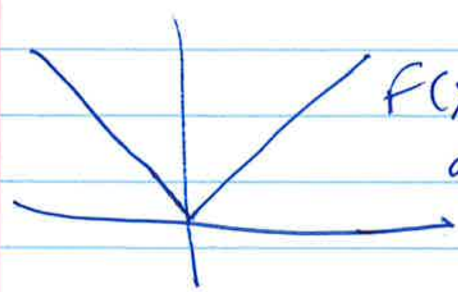
The simplest is $f'(a)$

Example if $f(t)$ is position of a car along a road at time t.

Then $F'(t)$ is the velocity of the car at time t.

Note $f'(t)$ is another function that can be evaluated at any point t.

The derivative may not exist, for example

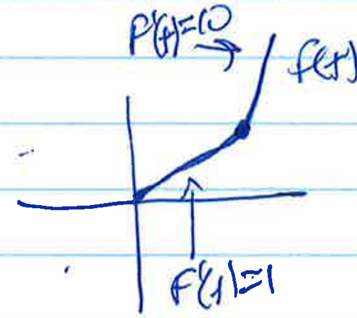


$f(x) = |x|$ does not have a unique tangent line at $x=0$

Another example

(If a car goes 1 mph for $t < 1$ and 10 mph for $t > 1$ how fast is it going at $t=1$?

Corner, no unique tangent line.



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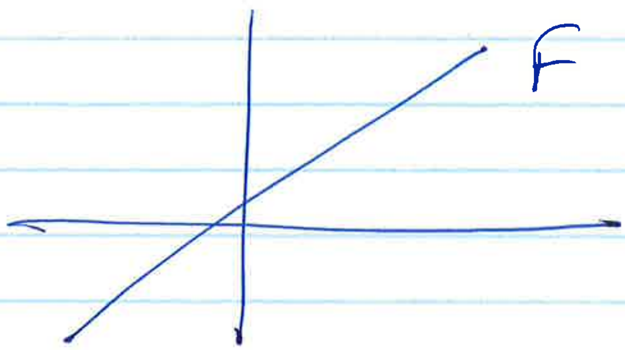
A car could not change its velocity abruptly though because it obeys Newton's law and the Force must be finite.

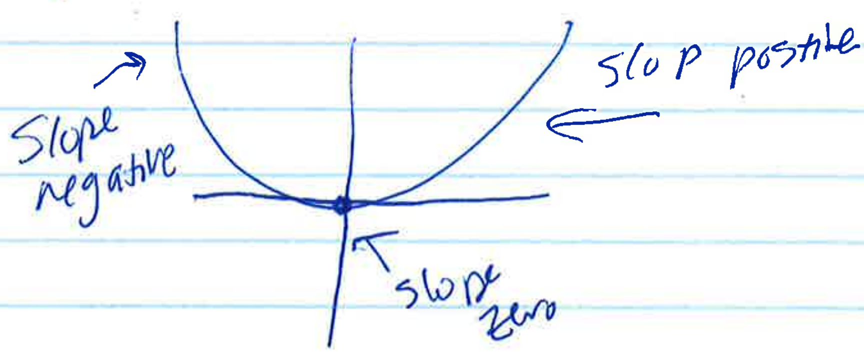
In fact we wont really encounter much Functions that have points where they are not differentiable in this class

A linear function has a constant slope

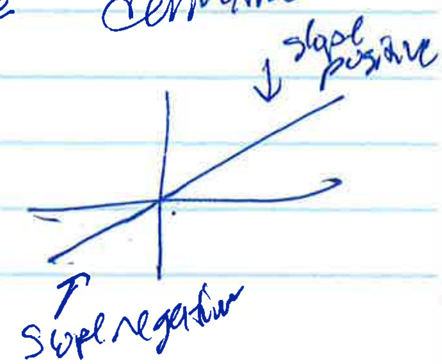


If the slope is m
what is the derivative?
what is the slope of the tangent line?

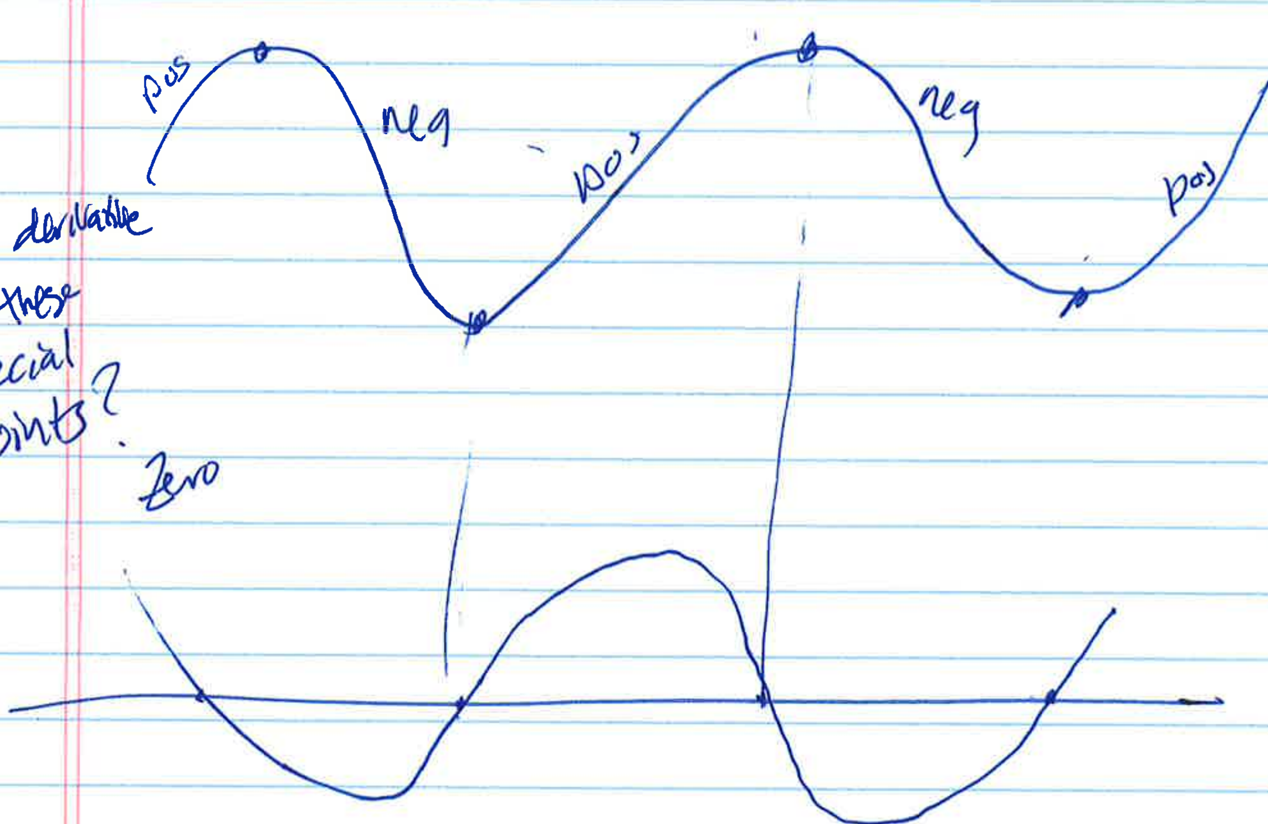
A parabola



derivative



If a function looks like



what derivative at these special points?

zero