Math 211
Summer 2014
Final Exam—4th Practice
6/26/14
Time Limit: 120 Minutes

Name (Print): Answers

This exam contains 8 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. Graphing calculators should not be needed, but they can be used to check your work. If you use a graphing calculator to find an answer you must write the steps needed to find the answer, without the calculator.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	20	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$\begin{aligned}
\frac{d}{dx}(x^n) &= nx^{n-1} \\
\frac{d}{dx}(cf(x)) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}(f(x)+g(x)) &= \frac{d}{dx}f(x) + \frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}(a^x) &= \ln(a)a^x \\
\frac{d}{dx}(\ln(x)) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g + fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\
\int x^n \, dx &= \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \\
\int \frac{1}{x} \, dx &= \ln(x) + C \\
\int e^{kx} \, dx &= \frac{1}{k}e^{kx} + C \\
\int (f(x)+g(x)) \, dx &= \int f(x) \, dx + \int g(x) \, dx \\
\int cf(x) \, dx &= c\int f(x) \, dx \\
E &= \left|\frac{p}{q}\frac{dq}{dp}\right|
\end{aligned}$$

1. (10 points) Suppose demand curve for a good is given by $p = 50 - 2q$, and the supply curve is given by $p = 1 + q$. Suppose the current price is set artificially \$1 above equilibrium. Find the consumer surplus.

$CS = \int_0^{q^+} f(q)\,dq - p^+q^+$

Find q^*: $50 - 2q^* = 1 + q^* \Rightarrow 49 = 3q^* \Rightarrow q^* = \frac{49}{3}$

Find p^*: $p^* = 1 + q^* = \frac{52}{3}$

Find p^+: $p^+ = p^* + 1$ (price 1 dollar above equilibrium)

$p^+ = 55/3$

Find q^+: Price limited by demand (above equilibrium)

$p^+ = 50 - 2q^+ \qquad 2q^+ = 50 - p^+$

$q^+ = \dfrac{50 - 55/3}{2} = 25 - \dfrac{55}{6} = \dfrac{150 - 55}{6}$

$= \dfrac{95}{6}$

$CS = \int_0^{95/6} (50 - 2q)\,dq - \dfrac{55}{3} \cdot \dfrac{95}{6}$

$50q + q^2 \Big|_0^{95/6} - \dfrac{55 \cdot 95}{3 \cdot 6}$

$= 50 \cdot \dfrac{95}{6} + \left(\dfrac{95}{6}\right)^2 - \dfrac{55 \cdot 95}{3 \cdot 6}$

$= 752.08$

2. (10 points) The elasticity of a good is $E = 0.9$. What is the effect of a 1% increase of price on the quantity demanded.

$$E = \left| \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \right| = .9 \qquad \frac{\Delta p}{p} = 1\%$$

$$\left| \frac{\Delta q}{q} \right| = 1\% \cdot .9 = .9\%$$

The effect of a 1% increase in price is a 0.9% <u>decrease</u> in quantity demanded.

3. (10 points) The demand for a product is given by $q = 200 - 4p^2$. Find the elasticity of demand when the price is \$6.

$$E = \left| \frac{p}{q} \frac{dq}{dp} \right|$$

$$p = 6$$

$$q = 200 - 4 \cdot 36 = 56$$

$$\frac{dq}{dp} = -8p = -8(6) = 48$$

4. (10 points) Find the present and future value of an \$15,000 per year income stream over 8 years, with a discount rate of 6%.

$$PV = \int_0^M S(t)\, e^{-rt}\, dt$$

$$= \int_0^8 15000\, e^{-0.06t}\, dt$$

$$= \frac{15000}{-0.06} e^{-0.06t} \Big|_0^8$$

$$= \frac{15000}{0.06}\left(1 - e^{-0.06 \cdot 8}\right)$$

$$= 95304.15 \qquad FV = PV \cdot e^{0.06 \cdot 8}$$

$$= 154018.60$$

5. (10 points) Find the average value of the function $f(x) = \frac{1}{x} + x$ within the interval [2, 4].

$$\frac{1}{b-a}\int_a^b F(x)\,dx = \frac{1}{2}\int_2^4 \left(\frac{1}{x} + x\right) dx$$

$$= \frac{1}{2}\left(\ln|x| + \frac{x^2}{2}\right)\Big|_2^4$$

$$= \frac{1}{2}\left(\ln(4) - \ln(2) + \frac{4^2}{2} - \frac{2^2}{2}\right)$$

$$= 3.3466$$

6. (10 points) The marginal cost function of a product is $C'(q) = 3q^2 - 100q + 200$. The fixed costs are \$300. Find the total cost to produce 50 items.

$$TC(b) = FC + \int_0^b MC(q)\,dq$$
$$= 300 + \int_0^{50} 3q^2 - 100q + 200$$
$$= 300 + q^3 - 50q^2 + 200q \Big|_0^{50}$$
$$= 300 + 50^3 - 50 \cdot 50^2 + 200 \cdot 50$$
$$= \$10300$$

7. (10 points) The relative rate of growth of a population~~,~~ between time t=2, and time t=5, is given by $r(t) = \frac{1}{x^2}$. Relate the population at time ~~4~~, to the population at time ~~1~~.

typos: 5 2

$$p(5) = e^{I} \cdot p(2)$$
$$I = \int_2^5 \frac{1}{x^2}\,dx = \int_2^5 x^{-2}\,dx = \frac{x^{-1}}{-1}\Big|_2^5$$
$$= -\frac{1}{5} + \frac{1}{2} = -\frac{2}{10} + \frac{5}{10} = \frac{3}{10}$$

The population at time 5 is $e^{3/10}$ times the population at time 2

8. (10 points) The local linear approximation is given by the formula

$$f(x) = f(a) + f'(a)(x - a).$$

For what type of function is the approximation exact? If the approximation is not exact, what characteristic of f makes the approximation better?

The approximation is exact for linear functions (if $f(x) = mx + b$)
The approximation is better if $f''(x)$ is small; also it is better for x close to a.

For x farther from a it is not as good unless the function is linear (equivalently $f''(x) = 0$)

9. (20 points) A bus company has fixed costs. For \$1 per ride the company attracts 100 passengers. With each additional \$0.10 the company loses 10 passengers (assume a linear demand eequation). How should the bus company set its price to maximize profits?

Fixed costs means profit is maximized where revenue is maximized

Revenue $= p \cdot q$ how do we find q? Demand Eqn

Demand linear: $q = mp + b$

$$m = \frac{\Delta q}{\Delta p} = \frac{-10}{.1} = -100$$

$$100 = -100 \cdot 1 + b$$
$$b = 200$$

Demand equation: $q = -100p + 200$

Revenue equation: $R(q) = pq = -100p^2 + 200p$

Critical Points of R: $-200p + 200 = 0$

$p = 1$

Maximum because $R''(p) = -200 < 0$

Company should leave ~~cost~~ price at \$1 to maximize profits.