Math 211
Spring 2015
Exam 2, Tuesday
3/31/15
Time Limit: 75 Minutes

Name (Print): Key

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	20	
2	10	
3	30	
4	30	
5	10	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$
\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1} \\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x) \\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x) \\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx} \\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x \\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x} \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
(fg)' &= f'g+fg' \\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2} \\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x) \\
\int x^n\ dx &= \frac{x^{n+1}}{n+1}+C, \quad n\neq -1 \\
\int \frac{1}{x}\ dx &= \ln(x)+C \\
\int e^{kx}\ dx &= \frac{1}{k}e^{kx}+C \\
\int \sin(kx)\ dx &= -\frac{1}{k}\cos(kx)+C \\
\int \cos(kx)\ dx &= \frac{1}{k}\sin(kx)+C \\
\int (f(x)+g(x))\ dx &= \int f(x)\ dx+\int g(x)\ dx \\
\int cf(x)\ dx &= c\int f(x)\ dx \\
E &= \left|\frac{p}{q}\frac{dq}{dp}\right|
\end{aligned}
$$

1. (20 points) Consider the function $f(x) = e^{2x} - x^3$.

(a) (5 points) What is average rate of change of f between $x = 2$ and $x = 3$?

$$\frac{f(3)-f(2)}{3-2} = \frac{e^6 - 27 - e^4 + 8}{1}$$

$$= e^6 - e^4 - 19 = 329.83$$

(b) (5 points) What is the instantaneous rate of change of f at $x = 2$?

$$f'(x) = 2e^{2x} - 3x^2$$

$$f'(2) = 2e^4 - 3 \cdot 4$$

$$= 2e^4 - 12 = 97.19$$

(c) (5 points) What is the relative rate of change of f at $x = 2$?

$$\frac{f'(x)}{f(x)} = \frac{97.19}{e^4 - 8} = \frac{2e^4 - 12}{e^4 - 8}$$

$$= 2.0257$$

(d) (5 points) Is f concave up, concave down, or neither, at $x = 2$?

$$f''(x) = 4e^{2x} - 6x =$$

$$f''(2) = 4e^4 - 12 = 206.39$$

Concave up

2. (10 points) Compute the derivative of $y = x^{x^2+1}$. Hint: we had a formula when x was in the exponent with constant base ($y = a^x$) and another formula when x was in the base with constant exponent $y = x^n$. In this case x is in both the exponent and base, and neither formula will be useful. Instead, take the log of both sides, use the chain rule and product rule and solve for $\frac{dy}{dx}$.

$$y = x^{x^2+1}$$

$$\ln y = \ln x^{x^2+1}$$

$$= (x^2+1)\ln x$$

$$\frac{1}{y}\frac{dy}{dx} = (x^2+1)\cdot\frac{1}{x} + 2x\cdot\ln x$$

$$\frac{dy}{dx} = y\left[\frac{x^2+1}{x} + 2x\ln x\right]$$

$$= x^{x^2+1}\left[\frac{x^2+1}{x} + 2x\ln x\right]$$

3. (30 points) Compute or estimate the following integrals, as specified:

(a) (10 points) Compute:

$$\int (x^2 + e^{3x} + 3\cos(2x))dx$$

$$= \frac{x^3}{3} + \frac{1}{3}e^{3x} + \frac{3}{2}\sin(2x) + C$$

(b) (10 points) Compute:

$$\int_0^4 e^{\frac{1}{2}t}dt$$

$$2e^{1/2t}\Big|_0^4 = 2e^2 - 2$$

✓ $= 12.778$

(c) (10 points) Estimate the following integral with a left-hand sum with $n = 3$ rectangles:

$$\int_1^7 (x^2e^{-x})dx$$

$\sum f(x_i)\Delta x$

$x=1 \quad x=3 \quad x=5$

$$= \boxed{1^2e^{-1}\cdot 2 + 3^2e^{-3}\cdot 2 + 5^2e^{-5}\cdot 2}$$

$$= \boxed{\frac{2}{e} + \frac{18}{e^3} + \frac{50}{e^5} = 1.969}$$

4. (30 points) The cost of producing a quantity q of an item is $C(q) = 3e^{q+1}$. The item sells for \$10.

(a) (10 points) What is the fixed cost?

$$C(0) = 3e^{0+1} = 3e = 8.155$$

(b) (10 points) What is the marginal cost at $q = 1$?

$$C'(q) = 3e^{q+1} = 3e^2$$

$$= 22.17$$

(c) (10 points) Find the critical points of the profit function. Classify each as a minimum or a maximum.

$$\pi = R - C = 10q - 3e^{q+1}$$

$$\pi'(q) = 10 - 3e^{q+1}$$

$$\pi'(q) = 0 \Rightarrow 3e^{q+1} = 10$$

$$e^{q+1} = \frac{10}{3}$$

$$q+1 = \ln\left(\frac{10}{3}\right)$$

$$q = \ln\left(\frac{10}{3}\right) - 1 = 0.2039$$

$$\pi''(q) = -3e^{q+1}$$

$$< 0$$

maximum

5. (10 points) A company's makes a product with fixed cost \$100, and marginal cost $e^q + 2$ dollars. What is the total cost of producing 10 units of the product.

$$C(10) = C(0) + \int_0^{10} C'(q)\,dq$$

$$= 100 + \int_0^{10} (e^q + 2)\,dq$$

$$= 100 + e^q + 2q \Big|_0^{10}$$

$$= 100 + e^{10} + 20 - 1 - 0$$

$$= 120 + e^{10} =$$

$$\$ 22146.46$$