Math 211
Spring 2014
Final Exam—3rd Practice
6/26/14
Time Limit: 120 Minutes

Name (Print): Answers

This exam contains 9 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, or notes, or cell phone. Calculator OK as long as it has no internet.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. Graphing calculators should not be needed, but they can be used to check your work. If you use a graphing calculator to find an answer you must write the steps needed to find the answer, without the calculator.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- Do not write in the table to the right.

Problem	Points	Score
1	15	
2	50	
3	10	
4	10	
5	10	
6	5	
Total:	100	

Useful derivative rules: here, a, c, k, and n are constants (i.e. do not depend on x) and are not necessarily integers.

$$
\begin{aligned}
\frac{d}{dx}\left(x^n\right) &= nx^{n-1}\\
\frac{d}{dx}\left(cf(x)\right) &= c\frac{d}{dx}f(x)\\
\frac{d}{dx}\left(f(x)+g(x)\right) &= \frac{d}{dx}f(x)+\frac{d}{dx}g(x)\\
\frac{d}{dx}\left(e^{kx}\right) &= ke^{kx}\\
\frac{d}{dx}\left(a^x\right) &= \ln(a)a^x\\
\frac{d}{dx}\left(\ln(x)\right) &= \frac{1}{x}\\
\frac{d}{dx}\sin(x) &= \cos(x)\\
\frac{d}{dx}\cos(x) &= -\sin(x)\\
(fg)' &= f'g+fg'\\
\left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2}\\
\frac{d}{dx}f(g(x)) &= f'(g(x))g'(x)\\
\int x^n\,dx &= \frac{x^{n+1}}{n+1}+C, \quad n\neq -1\\
\int \frac{1}{x}\,dx &= \ln(x)+C\\
\int e^{kx}\,dx &= \frac{1}{k}e^{kx}+C\\
\int (f(x)+g(x))\,dx &= \int f(x)\,dx+\int g(x)\,dx\\
\int cf(x)\,dx &= c\int f(x)\,dx\\
E &= \left|\frac{p}{q}\frac{dq}{dp}\right|
\end{aligned}
$$

1. (15 points) Find derivatives:

   (a) (5 points) $y = 3x^2 e^x$

   $6xe^x + 3x^2 e^x$ product Rule

   (b) (5 points) $y = \sin(\cos(x))$

   $\cos(\cos(x)) \cdot \sin(x)$ Chain Rule

   (c) (5 points) $y = \frac{1}{\sqrt{\ln(x)+e^x}}$

   $y = (\ln(x) + e^x)^{1/2}$

   $\frac{dy}{dx} = \frac{1}{2}(\ln(x) + e^x)^{-1/2} \cdot (\frac{1}{x} + e^x)$

2. (40 points) The demand curve is given by $p = 100 - q^2$. The supply curve is given by $p = 10 + q^2$

(a) (10 points) Find the equilibrium price and quantity.

Equilibrium

$$100 - q^2 = 10 + q^2$$

$$90 = 2q^2$$

$$q^2 = 45 \quad q^* = \sqrt{45}$$

$$p^* = 10 + q^{*2} = 10 + 45 = 55$$

Equilibrium price = \$55

Equilibrium quantity = $\sqrt{45} = 6.708$

(b) (10 points) Find the producer surplus when the market is in equilibrium.

$PS = p^* q^* + \int_0^{q^*} g(q)\,dq$ where g is supply curve

$$g(q) = 10 + q^2$$

$$PS = 55 \cdot \sqrt{45} + \int_0^{\sqrt{45}} (10 + q^2)\,dq$$

$$= 55 \cdot \sqrt{45} + 10q + \frac{q^3}{3} \Big|_0^{\sqrt{45}}$$

$$= 55 \cdot \sqrt{45} + 10 \cdot \sqrt{45} + \frac{45\sqrt{45}}{3}$$

$$= 536.6563$$

(c) (10 points) Find the consumer surplus when the market is in equilibrium.

$$CS = \int_0^{q^*} f(q)\,dq - p^* q^* = \int_0^{\sqrt{45}} (100 - q^2)\,dq - 55\sqrt{45}$$

$$= 100q - \frac{q^3}{3} - 55\sqrt{45}\Big|_0^{\sqrt{45}} =$$

$$100\sqrt{45} - \frac{45\sqrt{45}}{3} - 55\sqrt{45} = 201.246$$

(d) (10 points) Find the elasticity of demand at the equilibrium price (note formula on sheet)

$$E = \left|\frac{p}{q}\frac{dq}{dp}\right| \qquad p = 100 - q^2 \qquad p^* = 55$$

$$q^* = \sqrt{45}$$

Need $\frac{dq}{dp}$ Easiest to take derivative of demand equation with respect to P,

$$1 = -2q\frac{dq}{dp} \qquad \frac{dq}{dp} = \frac{-1}{2q} = \frac{-1}{2\sqrt{45}}$$

$$E = \left|\frac{55}{\sqrt{45}} \cdot \frac{-1}{2\sqrt{45}}\right| = \frac{55}{2 \cdot 45} = 0.61111$$

(e) (10 points) Find the total gain from trade when the price is fixed one dollar below its equilibrium value.

Price fixed below: use supply (if above use demand)

$p = 54 \quad 54 = 10 + q^2 \quad \bar{q} = \sqrt{44}$

$\int_0^{\sqrt{44}} (f(q) - g(q))\, dq = \int_0^{\bar{q}} (100 - q^2 - (10 + q^2))\, dq$

$= \int_0^{\sqrt{44}} (90 - 2q^2)\, dq = 90q + \frac{2}{3} q^3 \Big|_0^{\sqrt{44}}$

$= 90\sqrt{44} - \frac{2}{3} 44\sqrt{44} - 0 = 402.4171$

3. (10 points) The relative rate of growth of a population, between time t=1, and time t=2, is given by $r(t) = e^q + 1$. Relate the population at time 2, to the population at time 1.

$P(2) = P(1)e^I$

$I = \int_1^2 (e^q + 1)\, dq$

$= e^q + q \Big|_1^2$

$= e^2 + 2 - e - 1$

$= e^2 - e + 1 = 5.6707$

The population is $e^{5.67}$ times bigger at $t=2$

290.25 X bigger

4. (10 points) A bus company has fixed costs. For \$2 per ride the company attracts 400 passengers. With each additional \$0.10 the company loses 15 passengers (assume a linear demand eequation). How should the bus company set its price to maximize profits?

Profit is maximized when Revenue is maximized because bus company has fixed cost.

Revenue $= p \cdot q$ How do we get q?

Demand function linear

$q = mp + b$ (as in $y = mx + b$)

$m = \frac{\Delta q}{\Delta p}$ (as in $m = \frac{\Delta y}{\Delta x}$)

$= \frac{-15}{0.10} = -150$

$400 = (-150)2 + b \Rightarrow 400 + 300 = b \quad b = 700$

$q = -150p + 700$

$R = p \cdot q = p(-150p + 700) = -150p^2 + 700p$

$R' = -300p + 700 = 0 \Leftrightarrow p = \frac{700}{300} = \$2.33$

Maximum because $R'' = -300 < 0$

Company should raise price by \$0.33 to maximize profits.

5. (10 points) (a) (5 points) Find the tangent line to the graph of the function $f(x) = 2^x$ at $x = 1$.

(b) (5 points) Use the tangent line to approximate the function f at $x = 2$.

Function $f(x) = 2^x$

Point $x = 1$ $y = 2^1 = 2$

Tangent line through point $(1, 2)$

Slope? $f'(x) = \ln(2)\, 2^x$

$f'(1) = \ln(2) \cdot 2$

a) line $y = y_0 + m(x - x_0)$

$y = 2 + \ln(2) \cdot 2(x - 1)$

b) plug 2 in

$y = 2 + \ln(2)(2 - 1)$

$f(2) \approx 2 + \ln(2)$

6. (5 points) The local linear approximation is given by the formula

$$f(x) = f(a) + f'(a)(x - a).$$

Suppose the value of the function is 5, and its derivative is -6, both at $x = 3$. Approximate the function at $x = 3.1$. Approximate the function at $x = 10$. Which is likely to be a better approximation?

$f(3)=5$ use $a=3$

$f'(3)=6$

want $f(3.1)$. use $x=3.1$

then $x=10$

$f(3.1) = f(3) + f'(3)(3.1-3)$

$= 5 + 6(.1)$

$= 5.6$

$f(10) = 5 + 6\cdot(10)$

$= 65$

f(.1) is likely to be a better approximation

See graph