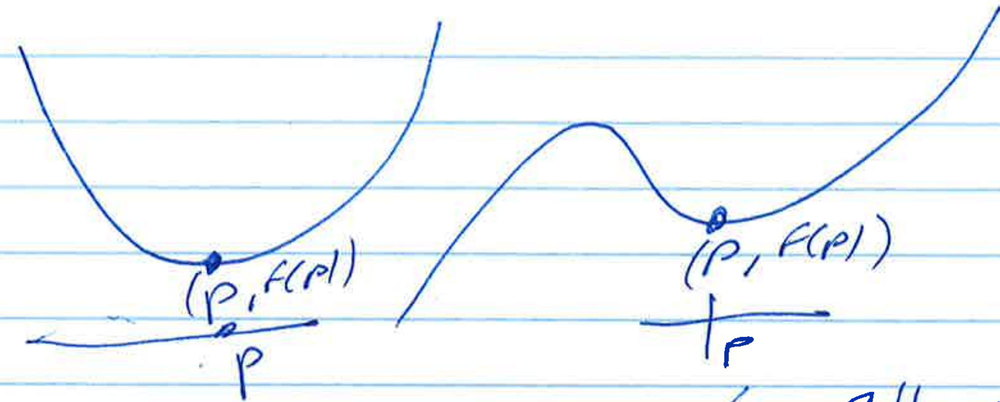
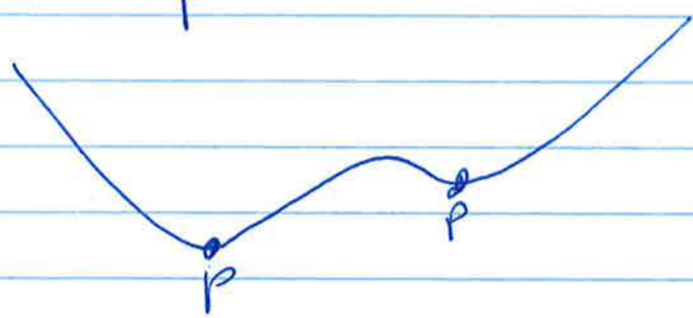
Math 211 - 2015S - Wb ~~Tues~~ Wed (Pg 1)

- Local Minimum - we want a definition of local minimum such that



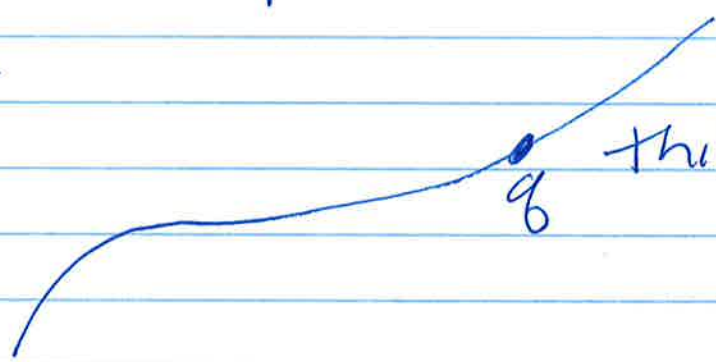
talking about both point in domain and point on graph

come back to this



all of these points are local minima

But



this point is not!

In all of the above points that we want to be a local minimum, no nearby point has a value less than the value at p

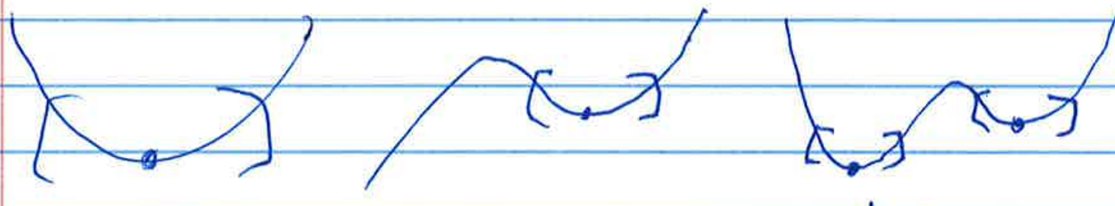
that is $f(p) \leq f(x)$ for x near p.

(domain not graph)

A mathematician might press you; what does it mean to be "near p"?.

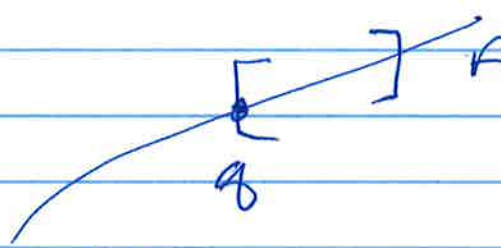
Answer: there exists an interval containing p in its interior such that

$f(p) \le f(x)$ in the interval



p has the lowest value in the interval (f has least value at p) even though outside of the interval, the function can go down further

Why must p be in the interior?

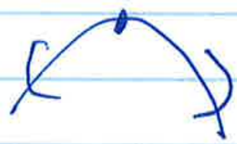


f has least value at q in interval but q is not a local minimum

Extend the endpoint so q in interior and q no longer least value in interval.

Local Maximum – likewise

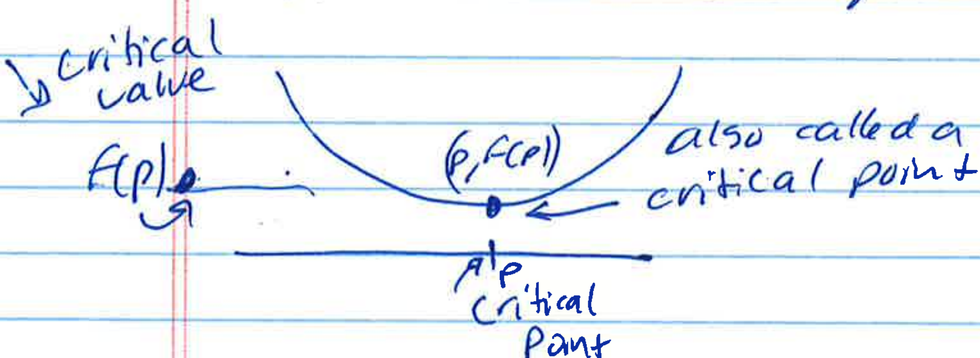
f has a local maximum at p
if $f(p) \geq f(x)$ for all points near p
(in an interval containing p ~~and~~ in interior

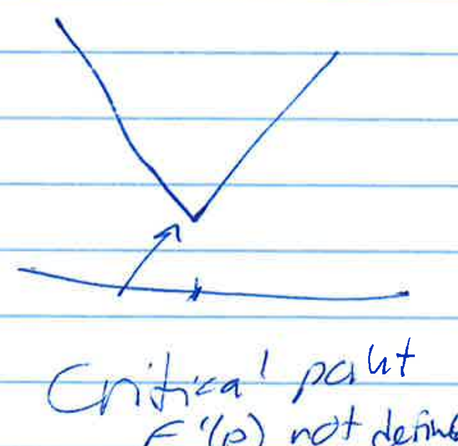


Critical point.

For any function f a point in the domain of f when $f'(p) = 0$ or $f'(p)$ is undefined is called a critical point

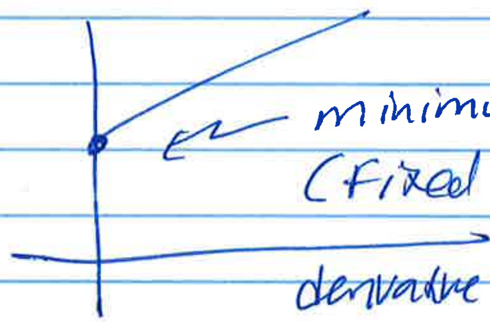
Also $(p, f(p))$ on the graph of a function is called a critical point





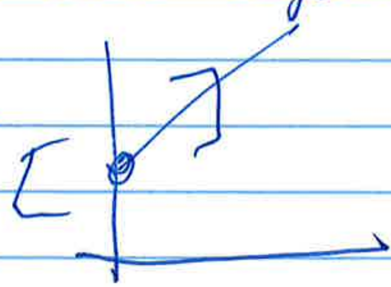
Result: If a function has a local min or local max at p then p is a critical point or p is the endpoint of an interval where f is defined

Remember cost function



minimum cost at quantity zero (fixed cost) even though derivative defined and not zero because its an endpoint of an interval when C(q) is defined,

Remember p had to be in the interior of an interval but the interval could extend beyond the domain



These sorts of things are studied in the branch of mathematics known as topology.

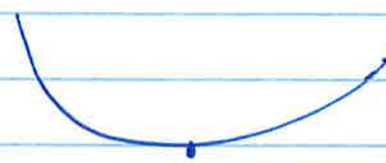
One more thing to be careful of

As I said if p is a min (or a max) then p is a critical point (or endpoint of domain)

However the converse is false

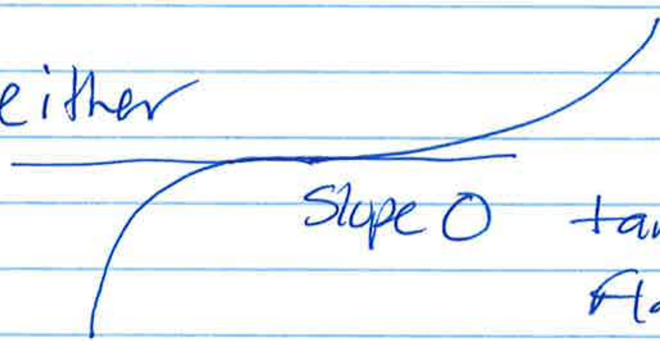
If p is a critical point, it may be

1) a minimum



2) a maximum

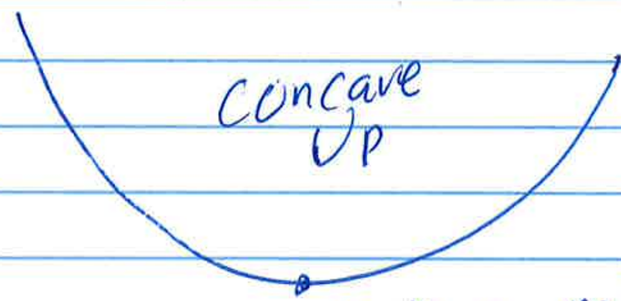
3) neither



Slope 0 tangent line flat but neither a min nor a max.

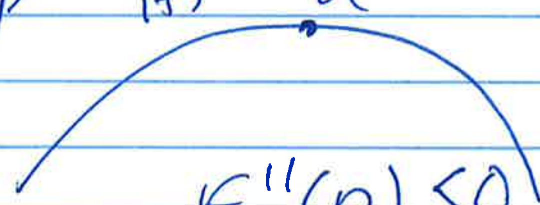
How do you tell?

If f is concave up at a critical point, its a minimum



$f''(p) > 0$ minimum

If f is concave down at a critical point p its a maximum

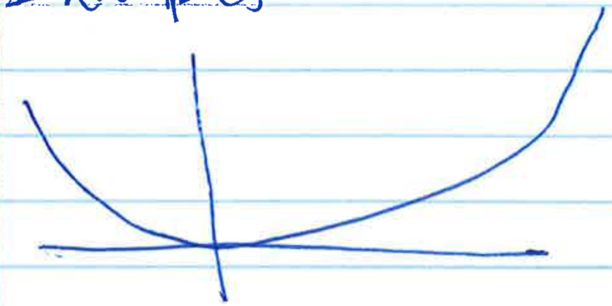


$f''(p) < 0$
maximum

If $f''(p) = 0$
p could be

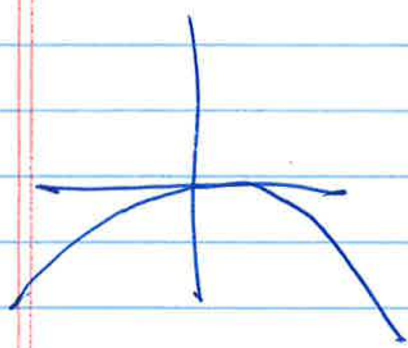
1) a minimum
2) a maximum
3) neither,

Examples



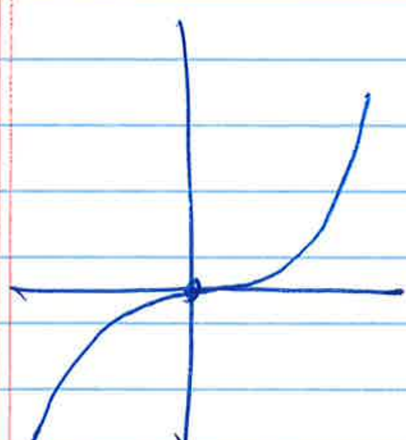
$f(x) = x^4$ $\quad f(0)=0$
$f'(x) = 4x^3$ $\quad f'(0)=0$
$f''(x) = 12x^2$ $\quad f''(0)=0$

Despite the fact $f''(0)=0$
f has a local ~~maxim~~ minimum at 0



$f(x) = -x^4$ $\quad f(0)=0$
$f'(x) = -4x^3$ $\quad f'(0)=0$
$f''(x) = -12x^2$ $\quad f''(0)=0$

Despite the fact $f''(0)=0$
f has a local max at 0.



$f(x) = x^3$ $\quad f(0)=0$
$f'(x) = 3x^2$ $\quad f'(0)=0$
$f''(x) = 6x$ $\quad f''(0)=0$

$f''(0)=0$ and f has neither a min nor a max at 0

Main idea: ~~you~~ Although you can tell
Critical point if $f'(0)=0$ and $f''(0) \neq 0$
If both are zero you can't tell.

Critical points $f'(p)=0$

if $f''(p)<0$ maximum

$f''(p)>0$ minimum

$f''(p)=0$ can't tell.

This is the procedure for finding local min/max

$\boxed{\text{Second derivative test}}$ (easiest but doesn't work if $f''(p)=0$)

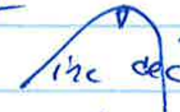
1. Compute $f'(x)$
2. Solve $f'(p)=0$ for p may be $(p_1, p_2, \dots p_n)$
3. Compute $f''(x)$
4. Plug in critical points for $f''(p_1)$ $f''(p_2)$ etc
5. Classify each according to $f''(p)>0$ min
   $f''(p)<0$ max

$\boxed{\text{First derivative test}}$ harder but works all the time

1. Compute $f'(x)$
2. Solve $f'(p)=0$ for p critical points
3. Also consider critical points where derivative is undefined (no problems in book require this)
4. On the intervals between critical points f will either be increasing $f'(p)>0$ or decreasing $f'(p)<0$

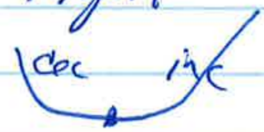
Pg 9

(a) If f is increasing on interval to left and decreasing on interval to right



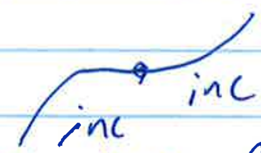
max

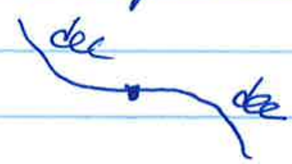
(b) If f is decreasing on interval to left an increasing ~~o~~ on interval to right



min

(c) If f is increasing or decreasing on both interval to left and right





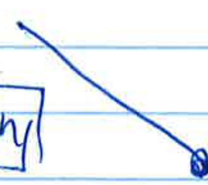
neither min nor max.

Finally You have to do one sided version of first derivative test for endpoints of domain

min

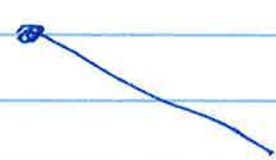
left endpoint
increasing to right

min



right endpoint ~~[illegible]~~
decreasing to left

max



left endpoint
~~[illegible]~~ decreasing to right

max

right endpoint
increasing to left

not emphasized in book