Math 211 - 20155 - W11 - Wed

Pg 1

Definite integral as an average.

Suppose we want to average a function. We could averge values

$$\text{Average} \approx \frac{f(t_1) + f(t_2) + f(t_3) + \cdots + f(t_n)}{n}$$

Let's say the t-values are equally spaced in the interval $0 \le t \le 24$

$\Delta t = 24/n$ so $n = 24/\Delta t$

$$\text{Average} \approx \frac{f(t_1) + f(t_2) + \cdots + f(t_n)}{24/\Delta t}$$

$$= \frac{f(t_1)\Delta t + f(t_2)\Delta t + \cdots + f(t_n)\Delta t}{24}$$

$$= \frac{1}{24} \sum_{i=1}^{n} f(t_i)\Delta t$$ ← Right handed Reimann Sum

The average of the function on the interval $0 \le t \le 24$ is the limit of the Riemann sum as $n \to \infty$ OR

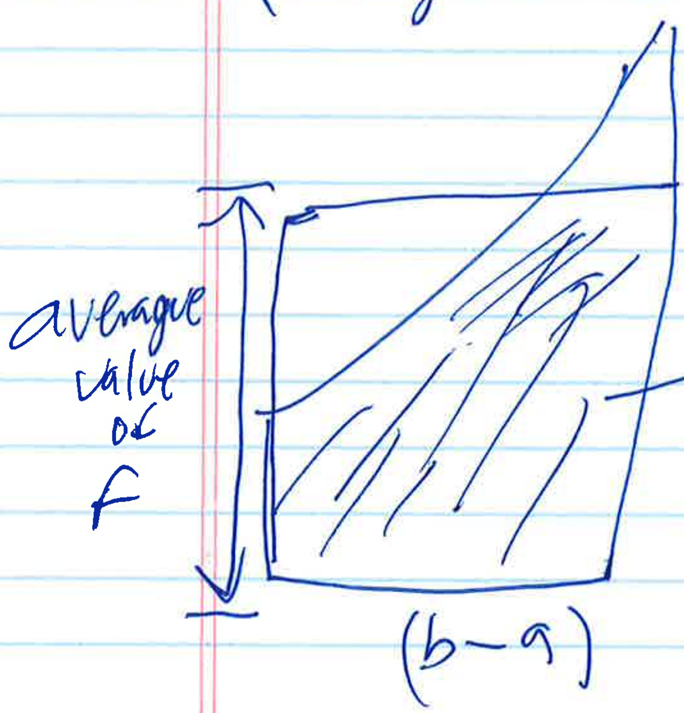
$$\frac{1}{24}\int_0^{24} f(\cancel{x})\,d\cancel{x}$$

more generally
Average value of f
on the interval from a to b

$$= \frac{1}{b-a}\int_a^b f(x)\,dx$$

Visualizing on graph

$$(\text{Average value of } f)(b-a) = \int_a^b f(x)\,dx$$



area of rectangle equals area under graph

The population of McAllen, TX can be modeled by the function

$$P = f(t) = (570)(1.037)^t$$

where P is in thousands of people and t is years since 2000

Predict the average population of McAllen between 2020 and 2040

$$\frac{1}{40-20}\int_{20}^{40} 570(1.037)^t$$

$$\frac{570}{20} \left. \frac{(1.037)^t}{\ln(1.037)} \right|_{20}^{40}$$

$$\frac{570}{20\ln(1.037)}\left[1.037^{40} - 1.037^{20}\right]$$

$$= 1732.81$$