Math 211-2015S-W3-Wed

Review

Present and Future Value

The future value B of a payment P today is $B = P(1+r)^t$

$$B = Pe^{rt}$$

The present value P of a payment B promised in the future is

$$P = B(1+r)^{-t}$$

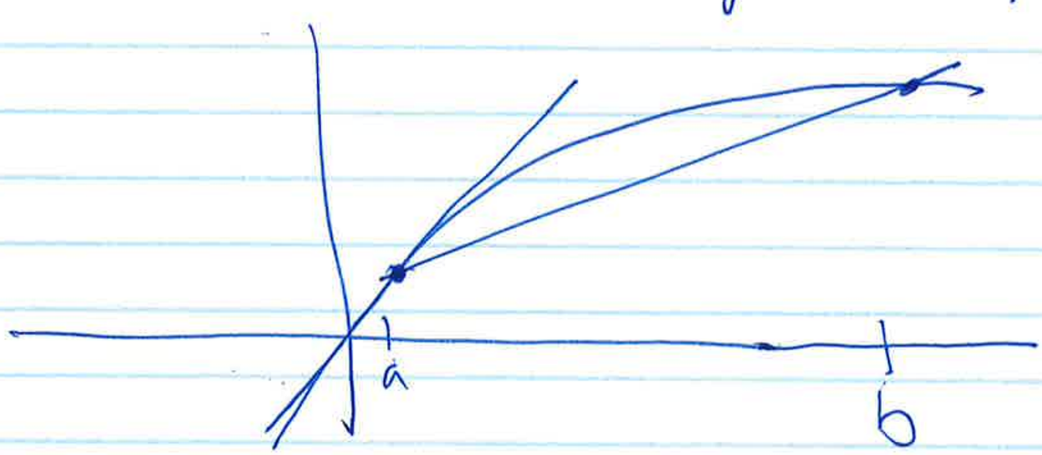
$$P = Be^{-rt}$$

These were same equations.

Average rate of change between two points

vs.

Instantaneous rate of change at one point

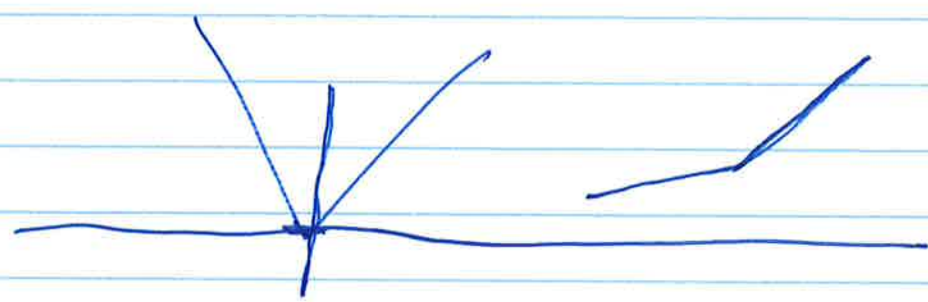


Math 211 - 2015S - W3 - Wed

Average rate ~~of~~ change between a and b
- slope of secant line
- difference equation

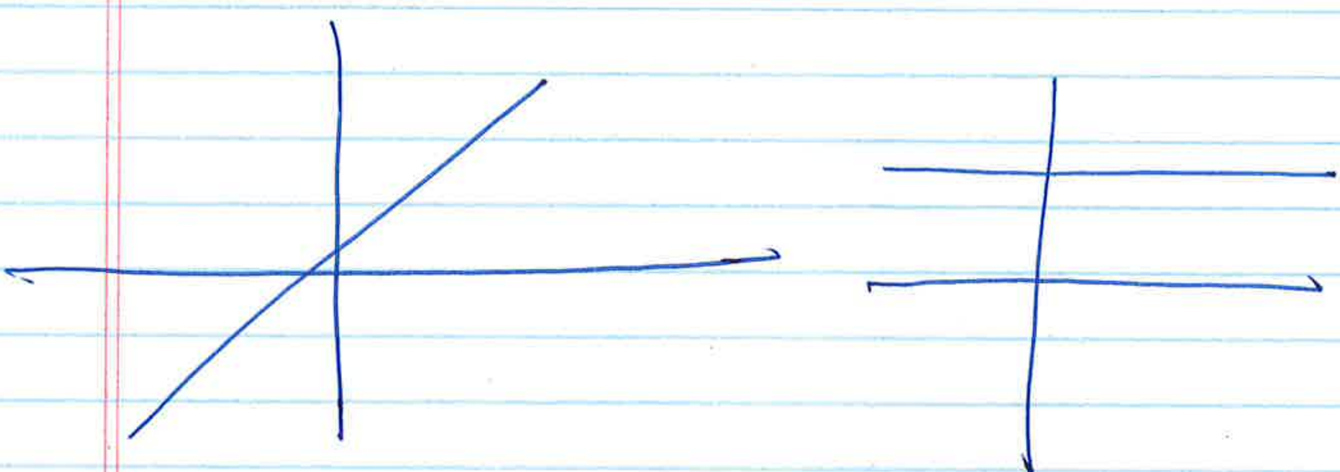
Instantaneous rate of change at a
- slope of tangent line
- limit of difference equation / limit of slopes of secant line / limit of average rate of change as $b \to a$
- derivative, $f'(a)$

Derivative ~~may~~ will not exist ~~wh~~ where function has a corner



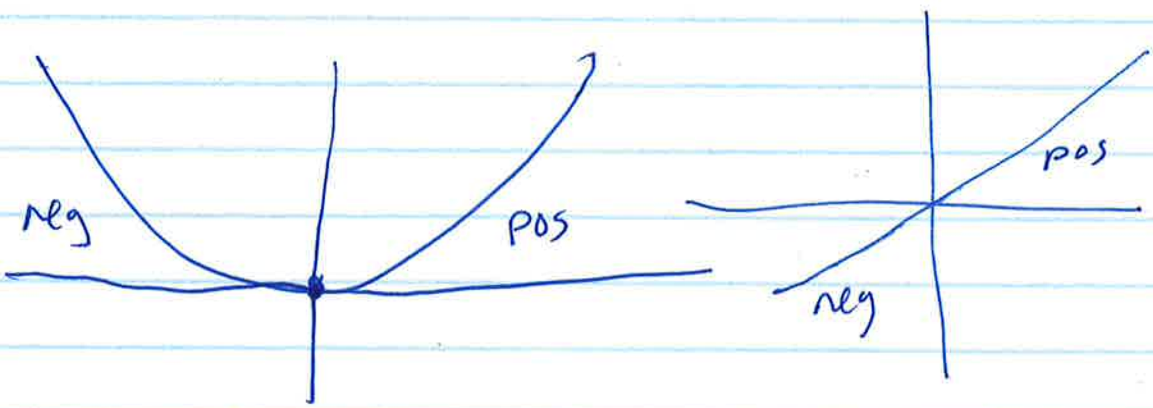
If function is position its derivative is velocity.

Derivative of a line with slope m

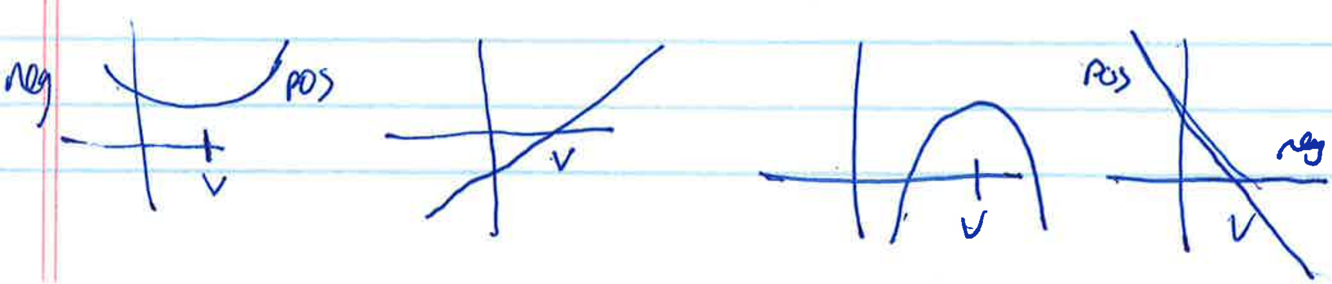


is a constant with value m

Derivative of a parabola



is a line ~~with horizontal~~ which crosses the x-axis at the vertex



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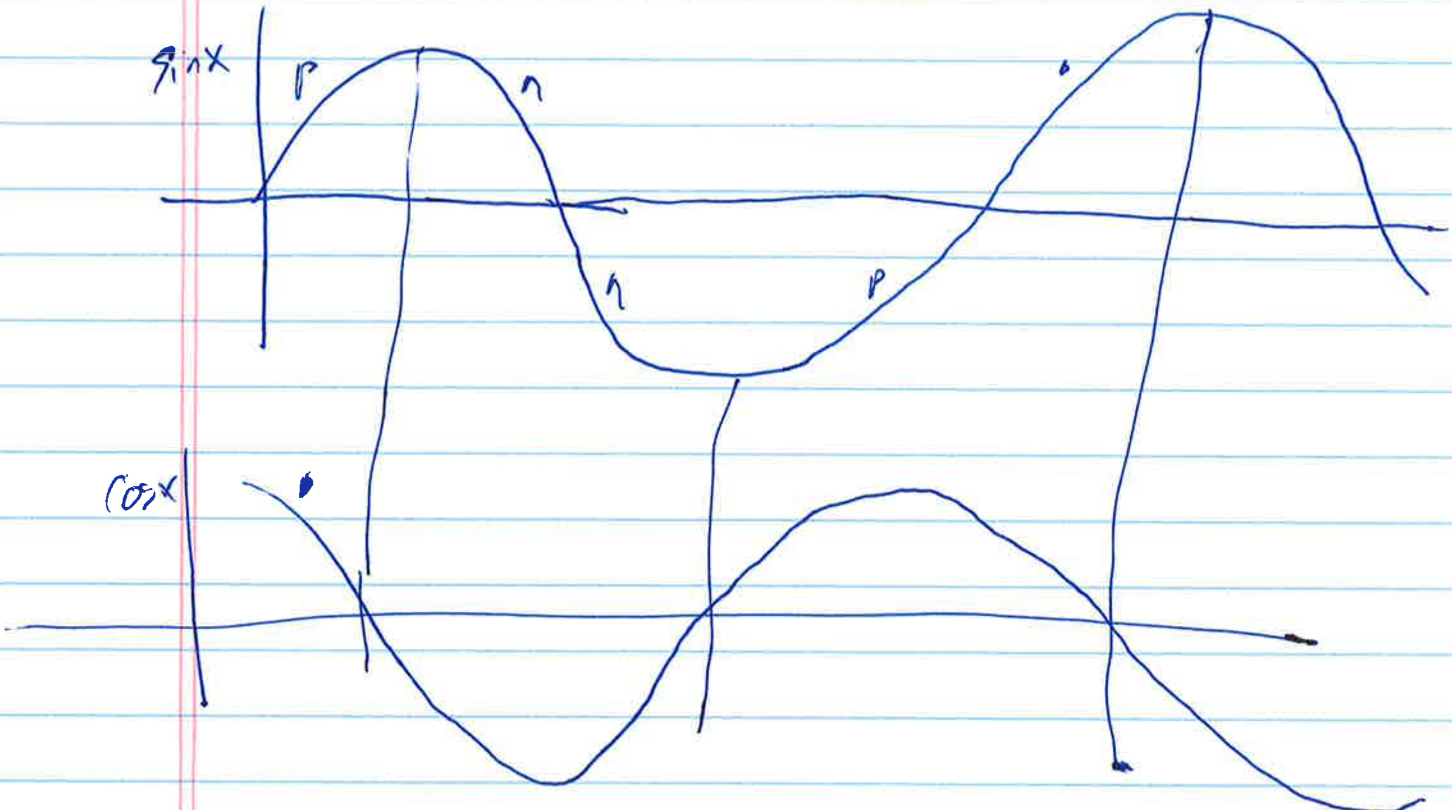
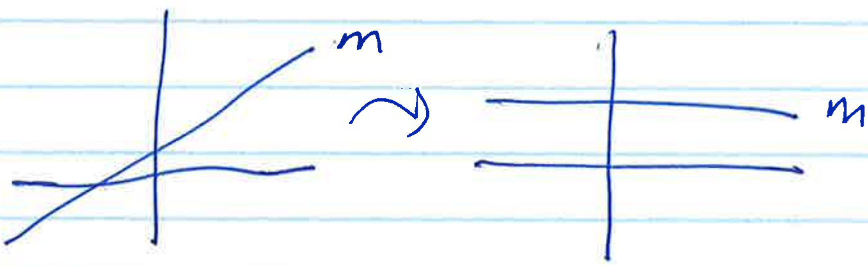
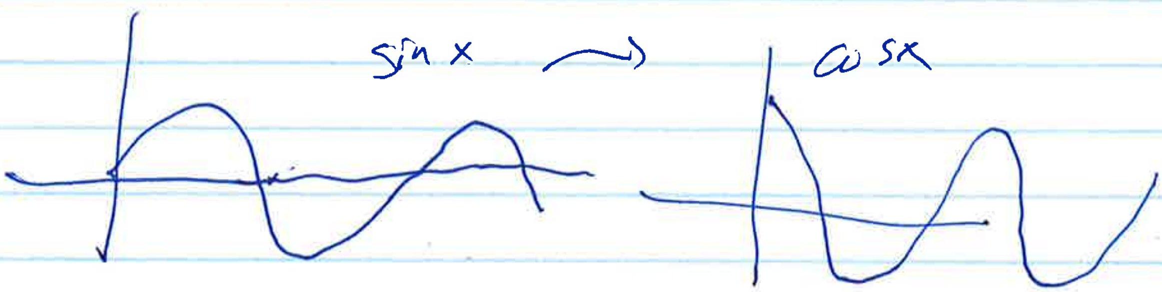


Once we have a function
We can make a new function

The value of this new function at x is the slope of the tangent line of the original function at x.





New

The new function is called the derivative of the old function.

The ~~new~~ derivative is called an operator which is a rule that takes a function as input and assigns a new function as output.

Let's review functions — from book definitions:

A function is a rule that takes a number as input and assigns a number as ~~output~~ output (such that each input has only one output).

↑ VLT

An operator is a rule that takes a function as input and assigns a ~~number~~ function as output (such that each input has only one output)
KLT

See symmetry?

Mathematically they're the same thing
In fact most definitions of functions
Cross out "number" inputs → outputs in def'n
left unspecified
to a mathematician an operator is just
a special kind of function.

Leibniz notation

The notation we've seen

There are two notations for the derivative

$y = f(x)$ function

$\boxed{\text{Derivative}}$ is $y = f'(x)$ we've seen this notation

$$f'(a) = \frac{f(b) - f(a)}{b - a} \quad \text{as } b \to a$$

$f(b) - f(a)$ is a small change in y

$b - a$ is a small change in x

New notation

$$f'(x) = \frac{\Delta y}{\Delta x}$$ as changes in x and y become small

$$= \frac{dy}{dx}$$

One interpretation

Confusing

dy is an infinitesimal change in y (or infinitely small) change in y)

dx an infinitesimal change in x

Another interpretation

least confusing

$\frac{dy}{dx}$ is just one symbol for

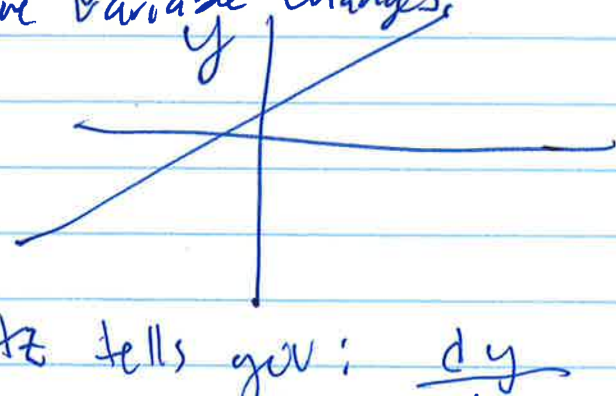
$f'(x)$

dy and dx don't mean anything on their own

Nice thing about Leibniz notation is that it tells you what variable is changing to get a change in y

$y = x^p$ if we want to find derivative of this function

the tangent line concerns changes in y as one variable changes



which variable goes on x-axis, is it x or p

Leibnitz tells you: $\frac{dy}{dx}$ versus $\frac{dy}{dp}$

The "numerator" tells you what function is changing, the "denominator" tells you what variable is changing to make the function change

Think of it as → But it is one symbol and the parts don't mean anything by themselves.

Units

$F'(x)$ or $\dfrac{dy}{dx}$ has

$$\frac{\text{Units of } y}{\text{Units of } x}$$

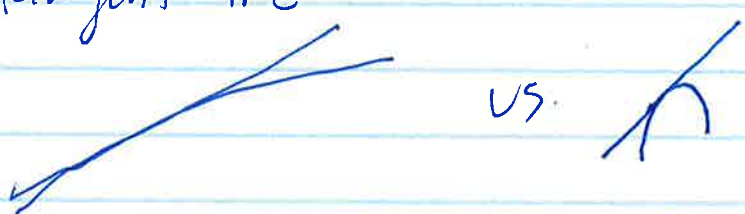
$$\text{Velocity} = \frac{\text{meter}}{\text{sec}}$$

meter ← position function

sec ← variable in position function

Marginal Cost

If the derivative of a function is not changing rapidly near a point then the function is a approximately the tangent line



then the slope of the tangent line (derivative) is approximately the amount the function (cost) changes when the variable (quantity)

Changes by 1

$$\frac{\text{rise}}{\text{run}} = \frac{m}{1} = m$$

The marginal cost is the slope (derivative) of the cost function

Which is approximately (if the derivative is not changing rapidly there) the cost for one more unit of good.