Stat 202 Exam 2 Second Practice Exam

1. (a) Explanatory: first test score (S_1)
   Response: second test score (S_2)
   because first explains second
   (b) Stat → Summary Stats → Correlation
   Select First, Second
   Correlation = 0.476
   (c) Stat → Regression → Simple Linear
   X variable: First, Y variable: Second $\Rightarrow$ Intercept 78.64, Slope 0.514

   $$S_2 = 0.514\, S_1 + 78.64$$

2. Correlation between X and Y = −1
   (perfect linear relationship, negative slope)

3. $Z = 3X + 4Y + 5$ $\mu_x = 1;\ \mu_y = 2;\ \sigma_x = 4;\ \sigma_y = 3$
   I should have said but didn't that
   X and Y are independent

   $$\mu_Z = 3\mu_x + 4\mu_y + 5$$
   $$= 3\cdot 1 + 4\cdot 2 + 5 = 16$$

   $$\sigma_Z^2 = 3^2\sigma_x^2 + 4^2\sigma_y^2 + 0$$
   $$= (9)(16) + (16)(9) = 228$$

   $$\sigma_Z = \sqrt{288} = 16.97$$

4 Z = load on elevator

X_i = weight of person i

$Z = X_1 + X_2 + X_3 + 50$, X_i's independent

$$\mu_Z = \mu_{X_1} + \mu_{X_2} + \mu_{X_3} + 50$$
$$= 175 + 175 + 175 + 50$$
$$= 575 \text{ mean}$$

$$\sigma_Z^2 = \sigma_{X_1}^2 + \sigma_{X_2}^2 + \sigma_{X_3}^2 + 0$$
$$= 3(10^2)$$
$$= 300$$

$$\left.\begin{aligned}\sigma_Z &= \sqrt{300} \\ &= 17.32\end{aligned}\right\}\text{standard deviation}$$

5. $\mu_{X_i} = 175$ $\sigma_{X_i} = 10$ $n = 10$

$$Z = X_1 + \cdots + X_{10}$$
$$\mu_Z = \mu_{X_1} + \cdots + \mu_{X_{10}}$$
$$= (10)(175)$$
$$= 1750 \text{ lbs}$$

$$W = \frac{1}{10}(X_1 + \cdots + X_{10})$$
$$\mu_W = \frac{1}{10}(\mu_{X_1} + \cdots + \mu_{X_{10}})$$
$$= \frac{1}{10} \cdot 10 \cdot 175$$
$$= 175 \text{ lbs}$$

$$\sigma_Z^2 = \sigma_{X_1}^2 + \cdots + \sigma_{X_{10}}^2$$
$$= 10 \cdot 100$$
$$= 1000$$

$$\sigma_W^2 = \frac{1}{10^2} \cdot \left(\sigma_{X_1}^2 + \cdots + \sigma_{X_{10}}^2\right)$$
$$= \frac{1}{10^2} 100 \cdot 10$$
$$= 10$$

$$\sigma_Z = \sqrt{1000} = 31.62$$

$$\sigma_W = \sqrt{10} = \frac{\sigma}{\sqrt{10}} = \frac{10}{\sqrt{10}} = \sqrt{10}$$

6 a) $z = \frac{x-\mu}{\sigma} = \frac{450-515}{41} = -1.585$

b) Stat → Calculators → Normal

Mean = 515

Std Dev = 41

$P(X \leq 450) = \underline{0.056}$

5.6th percentile

c) $P \leq \underline{582.4} = .95$

Score of 582.4 on 95th percentile

7. Success if 2 6 10 12 shows out of 20

$p(\text{success}) = 4/20 = .2$. Let X be # successes

This is binomial random variable

with $\begin{cases} p = .2 \\ n = 20 \quad (20 \text{ throws}) \end{cases}$

If $X \geq 5$ player wins

P player wins $= P(X \geq 5) = 37.03\%$
or $.3703$

P House wins $= 0.6296 = 1 - .3703$

House wins with prob 0.63, earns \$10
House loses with prob 0.37 earns \$10 − X
where X is the payoff X > 10 so house "earns" negative dollars, loses money

House makes/earns Y dollars (random)
Want $\mu_Y = 0$

Values of Y	10	10−X
Probability	.63	.37

$\mu_Y = (10)(.63) + (10 - X)(.37)$
$= 6.3 + 3.7 - .37X$
$= 10 - .37X = 0$

$.37x = 10 \quad x = 10/.37 = \$27.02$

House should pay \$27.02 to break even

Pg ~~5~~ 5

8a) probability of numbers 2 through 5 = 1/6
probability of 6 = .21
probability of 1 =

$1-\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+.21\right)=.1233$

b) $1/6 = .166667$

c) ~~Values of x~~

Values of X	1	2	3	4	5	6
Probabilities	.1233	1/6	1/6	1/6	1/6	.21

Mean $= 1\times.1233+\frac{2}{6}+\frac{3}{6}+\frac{4}{6}+\frac{5}{6}+(6)(.21)$
$=3.716667=\mu$

Variance ~~[illegible]~~ $=(1-\mu)^2(.1233)+(2-\mu)^2\cdot\frac{1}{6}$
$+(3-\mu)^2\frac{1}{6}+(4-\mu)^2\cdot\frac{1}{6}+(5-\mu)^2\cdot\frac{1}{6}$
$+(6-\mu)^2(.21)$

$=2.869$

Standard dev $=\sqrt{2.869}=1.694$

two dice are independent even if loaded

d) Z = sum of dots on two dice (loaded)
$\mu_Z=\mu_{X_1}+\mu_{X_2}=3.7167+3.7167=7.4333$
$\sigma_Z^2=\sigma_{X_1}^2+\sigma_{X_2}^2=2.869+2.869=5.7394$
$\sigma_Z=2.396=\sqrt{5.7394}$

9. Binomial Random Variable

X = number of zeros

$n=4$

$p=.1$ ← 1 digit in ten

$P(X \geq 1) = .34$

10 Binomial Random Variable

$n=5$

$p = 1/6 = .16667$ ← 1 face in 6

X be count of number of 6's

$P(X \geq 1) = 0.5981$

11. Choose a person at random.

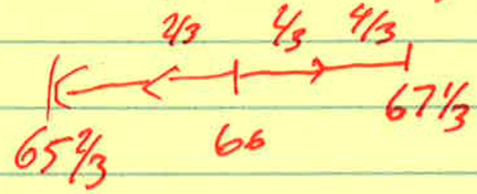
The event that the chosen person is female and the event that the person is over 75 are not independent because women live longer than men.

- Knowing a person is elderly means that person is more likely to be female.

- Therefore probabilities don't multiply.

12a) Sample mean = population mean
mean = 66 inches = $\mu_{\bar{x}}$

b) $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4/\sqrt{36} = 4/6 = \frac{2}{3} = .66667$

c) 

95% of sample means fall within
two standard deviations
lower limit = $65\frac{2}{3}$
upper limit = $67\frac{1}{3}$

d) Stat → Calculators → Normal

Mean 66 Std Dev .66667
$P(X \geq 66.1) = \underbrace{0.44038}_{\text{answer}}$

e) Want $2\sigma_{\bar{x}} = .1$
$\sigma_{\bar{x}} = .1/2 = .05 = 4/\sqrt{n}$

thus must have $\sqrt{n} = 4/.05 = 80$
$n = 6400$