Review

Integration by parts

Using the product Rule we derived the following formula

$$\int u\,dv = uv - \int v\,du$$

I plan to review this by working homework problems. Ask me.

New

Integration by substitution

Uses (not Product Rule) but Chain Rule.

This is facilitated by a "substitution" an inner function

For example

$$\int e^{x^2} \cdot 2x\,dx \qquad u = x^2$$
$$du = 2x\,dx$$

$$I = \int e^u\,du = e^u + c = e^{x^2} + C$$

put in term of x at end

$$\int (x^2+1)^5 \cdot 2x\,dx$$

$u = x^2+1 \qquad du = 2x\,dx$

$$= \int u^5\,du = \frac{1}{6}u^6 + C = \frac{1}{6}(x^2+1)^6 + C$$

$$\int \frac{1}{x^2+4} \cdot 2x\,dx$$

$u = x^2+4$

$du = 2x\,dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C$$

$$= \ln(x^2+4) + C$$

$$\int t e^{(t^2+1)} dt$$

$u = t^2 + 1$
$du = 2t\,dt$

mult by ½

need a 2

$$= \frac{1}{2}\int e^{(t^2+1)} 2t\,dt$$

$$= \frac{1}{2}\int e^{u} du$$

$$= \frac{1}{2} e^{u} + C = \frac{1}{2} e^{(t^2+1)} + C$$

Warning: this only works for constants. If we need a t

$$\int e^{(t^2+1)} dt$$

We'd like to do $\frac{1}{2t}\int 2t\,dt\, e^{t^2+1}$

But this is wrong: We can't move $1/2t$ outside integral. That integral has no solution.

Pg 4

$$\int x^3\sqrt{x^4+5}\,dx$$

$u = x^4+5$
$du = 4x^3dx$

$$\frac{1}{4}\int 4x^3dx\,\sqrt{x^4+5}$$

Only because a constant

$$\frac{1}{4}\int u^{1/2}du$$

$$= \frac{1}{4}\,\frac{2}{3}u^{3/2}+c$$

$$\frac{1}{6}(x^4+5)^{3/2}+c$$