Math 211-2015S-W4-Tues

Review

The second derivative

The derivative of a function is itself a function, so it can have a derivative, too.

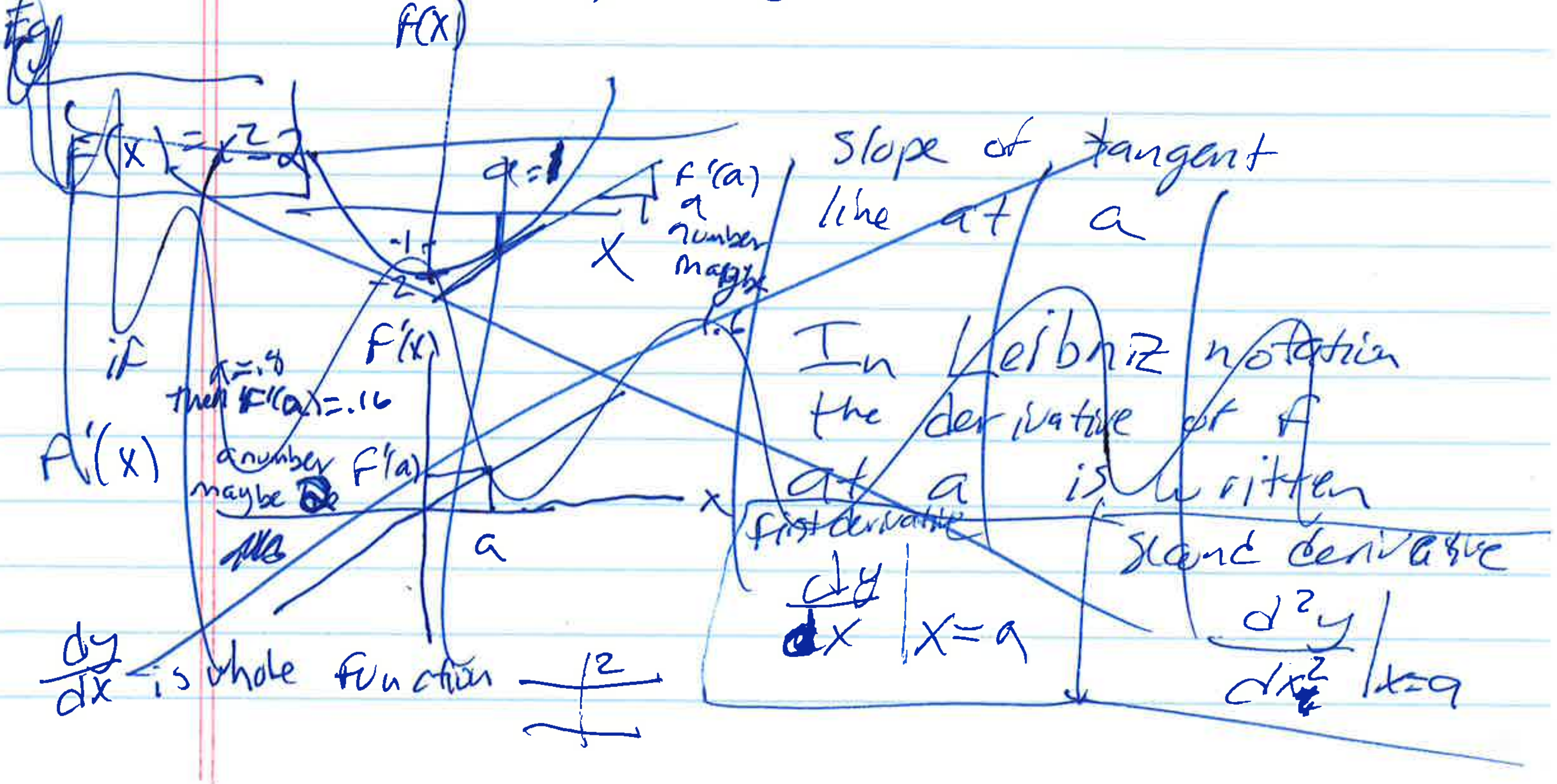
The derivative of a derivative of a function is called the second derivative of the function.

function: $y=f(x)$, $f(x)$

DENOTED: derivative: $f'(x)$ or $\frac{dy}{dx}$

second derivative: $f''(x)$ or $\frac{d^2y}{dx^2}$ or $\frac{d}{dx}\left(\frac{dy}{dx}\right)$

Note Newton notation has a more natural way of denoting a point where the derivative is taken, not just the whole function



slope of tangent line at a

$\frac{dy}{dx}$ is whole function

In Leibniz notation the derivative of f at a is written

first derivative

$$\frac{dy}{dx}\Big|_{x=a}$$

second derivative

$$\frac{d^2y}{dx^2}\Big|_{x=a}$$

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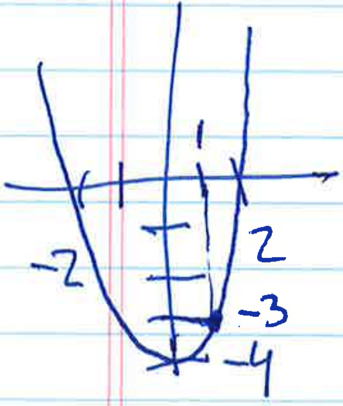
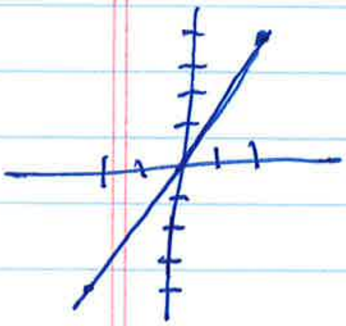
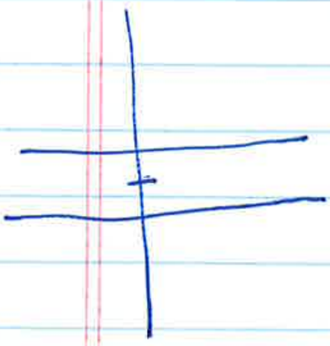
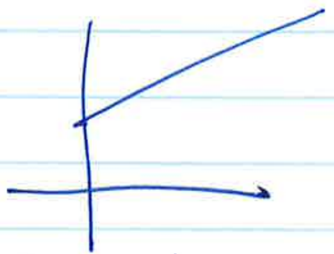
	Newton	Leibniz
 	$F(x) = x^2 - 4$ parabola opens up $F(0) = -4$ $F(2) = 0$ $F(-2) = 0$	$F(x) = y = x^2 - 4$ $y\vert_{x=0} = -4$ Usually use Newton but to be consistent $y\vert_{x=2} = 0$ $y\vert_{x=-2} = 0$
we'll show later → 	$F'(x) = 2x$ line through origh, Slope 2 $F'(0) = 0$ $F'(2) = 4$ $F'(-2) = -4$	$\frac{dy}{dx} = 2x$ $\frac{dy}{dx}\Big\vert_{x=0} = 0$ $\frac{dy}{dx}\Big\vert_{x=2} = 4$ $\frac{dy}{dx}\Big\vert_{x=-2} = -4$
	$F''(x) = 2$ Constant function value 2 $F''(0) = 2$ $F''(2) = 2$ $F''(-2) = 2$	$\frac{d^2y}{dx^2} = 2$ $\frac{d^2y}{dx^2}\Big\vert_{x=0} = 2$ $\frac{d^2y}{dx^2}\Big\vert_{x=2} = 2$ $\frac{d^2y}{dx^2}\Big\vert_{x=-2} = 2$

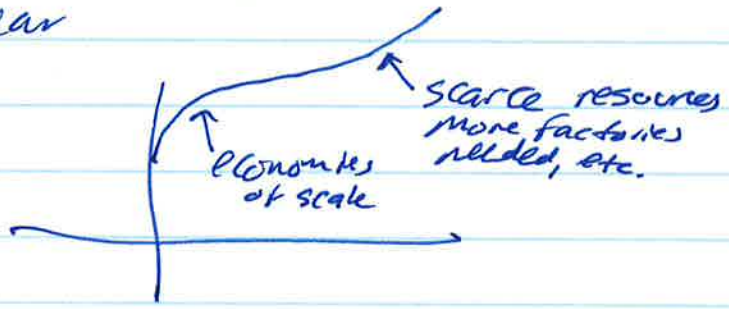
If $F' > 0$ then F increasing	If $f' < 0$ then f decreasing
If $F'' > 0$ then F' increasing and F concave up	If $f'' < 0$ then f' decreasing and f concave down

The above holds on interval where the (second) derivative has specified sign.

Marginal Cost - Derivative of cost function depends on q if cost function is curved/nonlinear



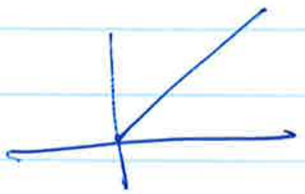
typical linear cost funct

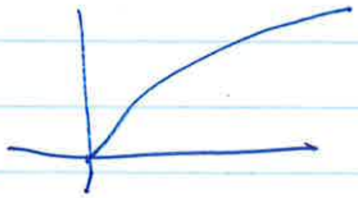


typical non linear cost function

Marginal Revenue - Derivative of Revenue function

price when linear





models market glutting.

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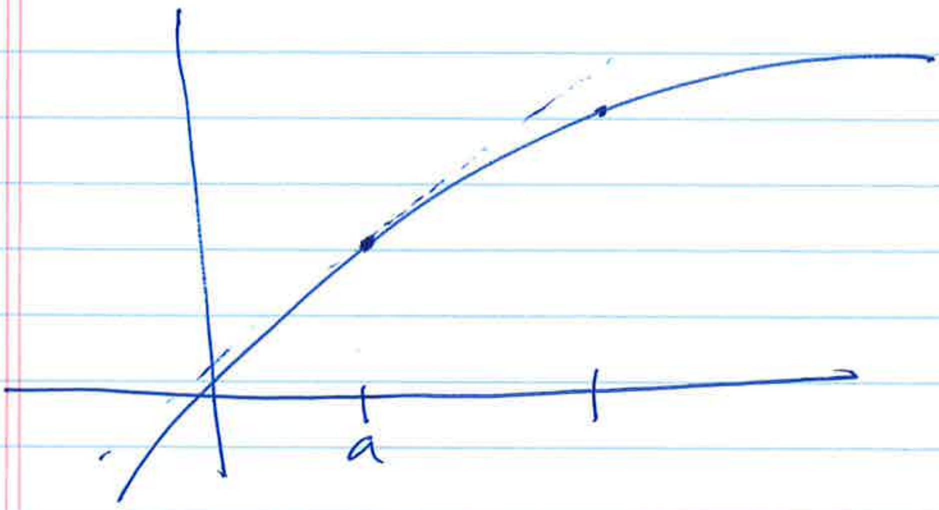
Local Linear Approximation



We Know a, $f(a)$, $f'(a)$ and x

We want to approximate $f(x)$ as "$F(x)$"

(non standard notation) {A standard notation would be $\widehat{F(x)}$ or better: $\hat{y}$

Draw the tangent line to f at a

It is the line through $(a, f(a))$ with slope $f'(a)$.

What point has horizontal coordinate x?

Call it "$F(x)$" $= \hat{y}$ ↗ "$F(x)$".

Rise / Run $\frac{\hat{y} - f(a)}{x - a} = f'(a)$

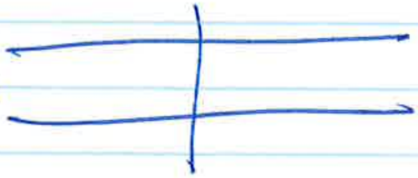
or "$F(x)$" $= \hat{y} = f(a) + f'(a)(x-a)$

New

How do you find derivatives?

Derivative of a constant function

$f(x) = k$

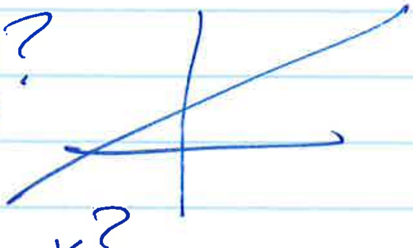


What's the graph?
What's the slope?

$f'(x) = 0$

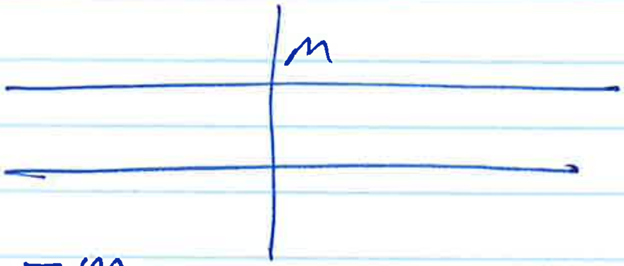
Derivative of a linear function

$y = mx + b \quad f(x) = mx + b$



What's the graph?
What's the slope?

Does it depend on x?



$f'(x) = m$

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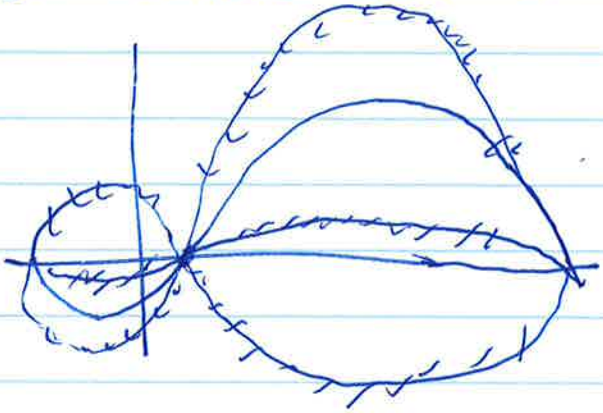
Derivative of a constant multiple

Multipling by a constants shrinks or stretches graph or reflects about x-axis



It turns out that the slope changes by the same multiplicative factor! You can kind of see this in graph (above)

Formula $$\frac{d}{dx}\Big(c\,f(x)\Big) = c\,\frac{d}{dx}\Big(f(x)\Big)$$

Example $$\frac{d}{dx}\Big(5\underbrace{(3x+1)}_{\text{Slope }3}\Big) = 5\frac{d}{dx}\underbrace{(3x+1)}_{=5\cdot 3}$$

$$= \frac{d}{dx}(15x+1) = 15$$

Derivative of sum and Difference

What's the derivative of

Example $y_1 = 3X$ and $y_2 = 2X$

$\frac{dy_1}{dx} = 3$ $\frac{dy_2}{dx} = 2$

$y_1 + y_2 = 5X$

$$\frac{d(y_1+y_2)}{dx} = 5 = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

This holds for all functions!

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

$$\# \quad = f'(x) + g'(x)$$

$$\frac{d}{dx}\left[f(x)-g(x)\right] = f'(x) - g'(x)$$

$$= \frac{d}{dx}\left[f(x)\right] - \frac{d}{dx}\left[g(x)\right]$$

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Derivatives of Powers of X

variable

constant power } not exponentials must use different formula coming soon

$$\frac{d}{dx}\left(x^2\right) = 2x$$

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

$$\frac{d}{dx}\left(x^4\right) = 4x^3$$

$$\frac{d}{dx}\left(x^n\right) = n\,x^{n-1}$$

n can be positive / negative / zero
integer / fraction } still works

$$\frac{d}{dx}\left(x^{1/2}\right) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}\,x^0 = \frac{d}{dx}(1) = 0 \cdot x^{-1} = 0$$

$$\frac{d}{dx}\left(x^{-1}\right) = -1\,x^{-2} = \frac{-1}{x^2}$$

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Derivatives of polynomials

$$\frac{d}{dx}\left[5x^4+3x^3+2x^2+6x+2\right]$$

take it term by term. The exponent gets multiplied by the coefficint to make the new coefficient and then te exponent gets reduced by 1

$$= 20x^3+9x^2+4x+6+0$$

Parabolas have equation

$$y = ax^2+bx+c$$

So

$$\frac{dy}{dx} = \underbrace{2ax+b}$$

a line! ~~just like~~

I told you so!

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Derivatives of Exponentials

We had derivatives of power functions

$f(x) = x^n$ ← constant

← variable

answer

$f'(x) = n x^{n-1}$

what if the variable is in the exponent and the base is constant

$f(x) = e^x$ ← variable

← constant

$f'(x) = e^x$ ● function equals its derivative

$f(x) = a^x$ diffrent base

$f'(x) = (\ln a) a^x$ if $a = e$ $\ln e = 1$

formula comes out nicer

I told you so!

$f(x) = e^{kx}$

$f'(x) = k e^{kx}$

$f(x) = \ln(x)$

$f'(x) = 1/x$

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- The process of starting with a function f and finding the derivative is sometimes called
    - "taking the derivative" of f

  or

    - "differentiating" f
- What you are doing is finding the slope of the tangent line at each point x as a function of x.
- To find the second derivative, you take the derivative twice.